Digital Design & Computer Arch.

Lecture 6: Sequential Logic Design

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ETH Zürich

Spring 2020

6 March 2020

We Are Almost Done with This

- Building blocks of modern computers
  - Transistors
  - Logic gates
- Combinational circuits
- Boolean algebra
- How to use Boolean algebra to represent combinational circuits
- Minimizing logic circuits

Agenda for Today and Next Week

- Today
  - Wrap up Combinational Logic and Circuit Minimization
  - Start (and finish) Sequential Logic
- Next week
  - Hardware Description Languages and Verilog
    - Combinational Logic
    - Sequential Logic
  - Timing and Verification

Assignment: Required Lecture Video

- Why study computer architecture?
- Why is it important?
- **Future Computing Architectures**

- **Required Assignment**
  - **Watch** Prof. Mutlu's inaugural lecture at ETH and understand it
  - https://www.youtube.com/watch?v=kgiZlSOcGFM

- **Optional Assignment – for 1% extra credit**
  - **Write a 1-page summary** of the lecture and email us
    - What are your key takeaways?
    - What did you learn?
    - What did you like or dislike?
    - Submit your summary to Moodle – Deadline: April 1

Extra Assignment: Moore's Law (I)

- **Paper review**
- G.E. Moore. "Cramming more components onto integrated circuits," Electronics magazine, 1965

- **Optional Assignment – for 1% extra credit**
  - **Write a 1-page review**
  - Upload PDF file to Moodle – Deadline: April 1

- I strongly recommend that you follow my guidelines for (paper) review (see next slide)

Extra Assignment 2: Moore's Law (II)

- **Guidelines on how to review papers critically**
  - Guideline slides: pdf ppt
  - Video: https://www.youtube.com/watch?v=tOL6FANAJ8c
  - Example reviews on "Main Memory Scaling: Challenges and Solution Directions" (link to the paper)
    - Review 1
    - Review 2
  - Example review on "Staged memory scheduling: Achieving high performance and scalability in heterogeneous systems" (link to the paper)
    - Review 1

Assignment: Required Readings

- Combinational Logic
  - P&P Chapter 3 until 3.3 + H&H Chapter 2
- Sequential Logic
  - P&P Chapter 3.4 until end + H&H Chapter 3 in full
- Hardware Description Languages and Verilog
  - H&H Chapter 4 in full
- Timing and Verification
  - H&H Chapters 2.9 and 3.5 + (start Chapter 5)

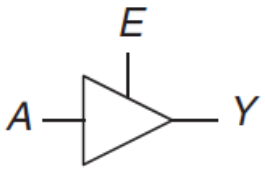
- By the end of next week, make sure you are done with
  - **P&P Chapters 1-3 + H&H Chapters 1-4**

Wrap-Up Combinational Logic Circuits and Design

Recall: Tri-State Buffer

- A tri-state buffer enables gating of different signals onto a wire

Tristate Buffer



E	A	Y
0	0	Z
0	1	Z
1	0	0
1	1	1

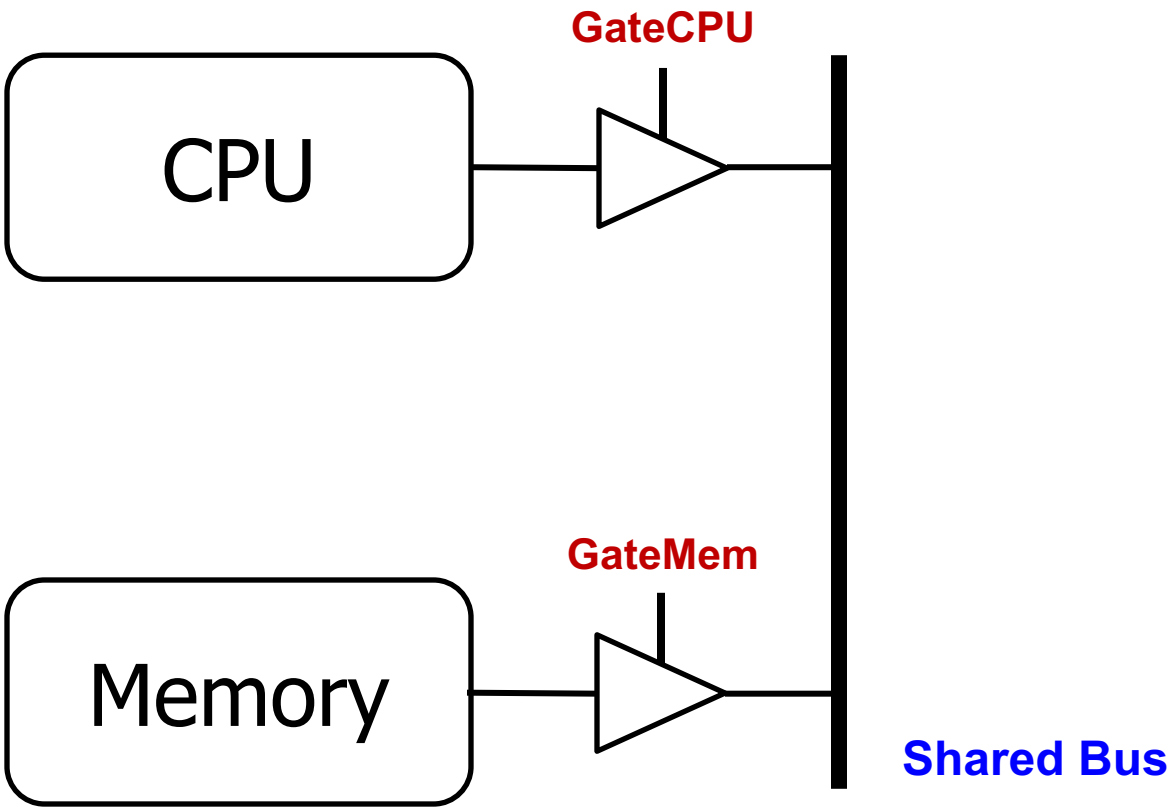
Figure 2.40 Tristate buffer

- Floating signal (Z): Signal that is not driven by any circuit
  - Open circuit, floating wire

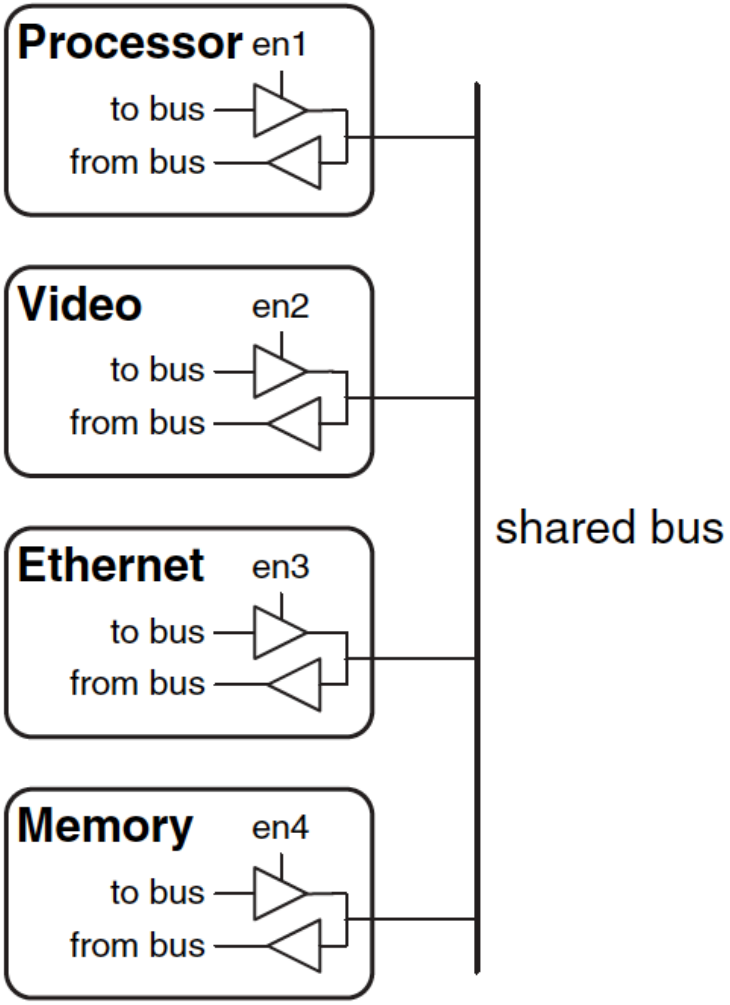
Recall: Example: Use of Tri-State Buffers

- Imagine a wire connecting the CPU and memory
  - At any time only the CPU or the memory can place a value on the wire, both not both
  - You can have two tri-state buffers: one driven by CPU, the other memory; and ensure at most one is enabled at any time

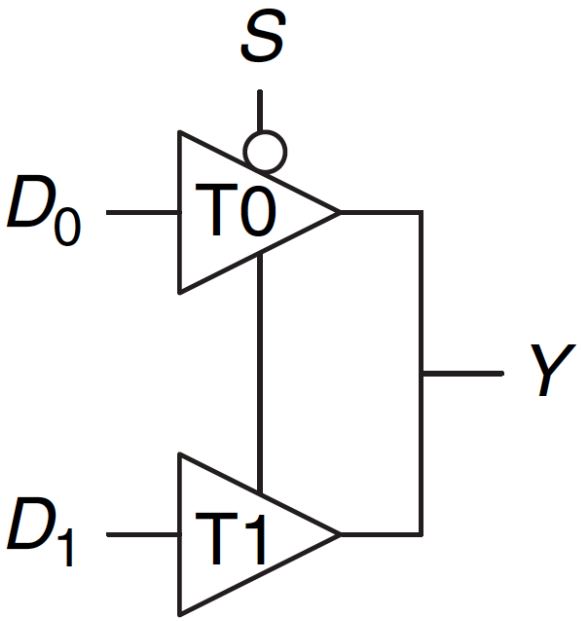
Recall: Example Design with Tri-State Buffers



Recall: Another Example



Multiplexer Using Tri-State Buffers



$$Y = D_0\bar{S} + D_1 S$$

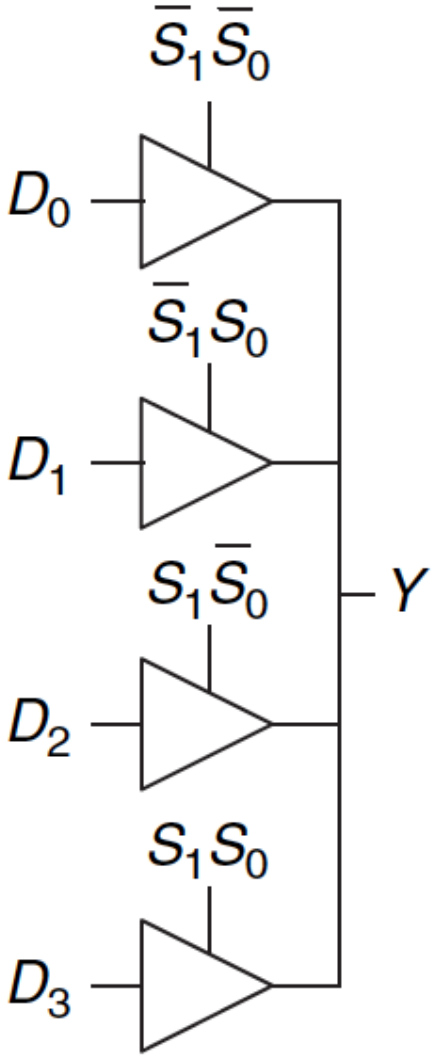


Figure 2.56 Multiplexer using tristate buffers

Aside: Logic Using Multiplexers

- Multiplexers can be used as lookup tables to perform logic functions

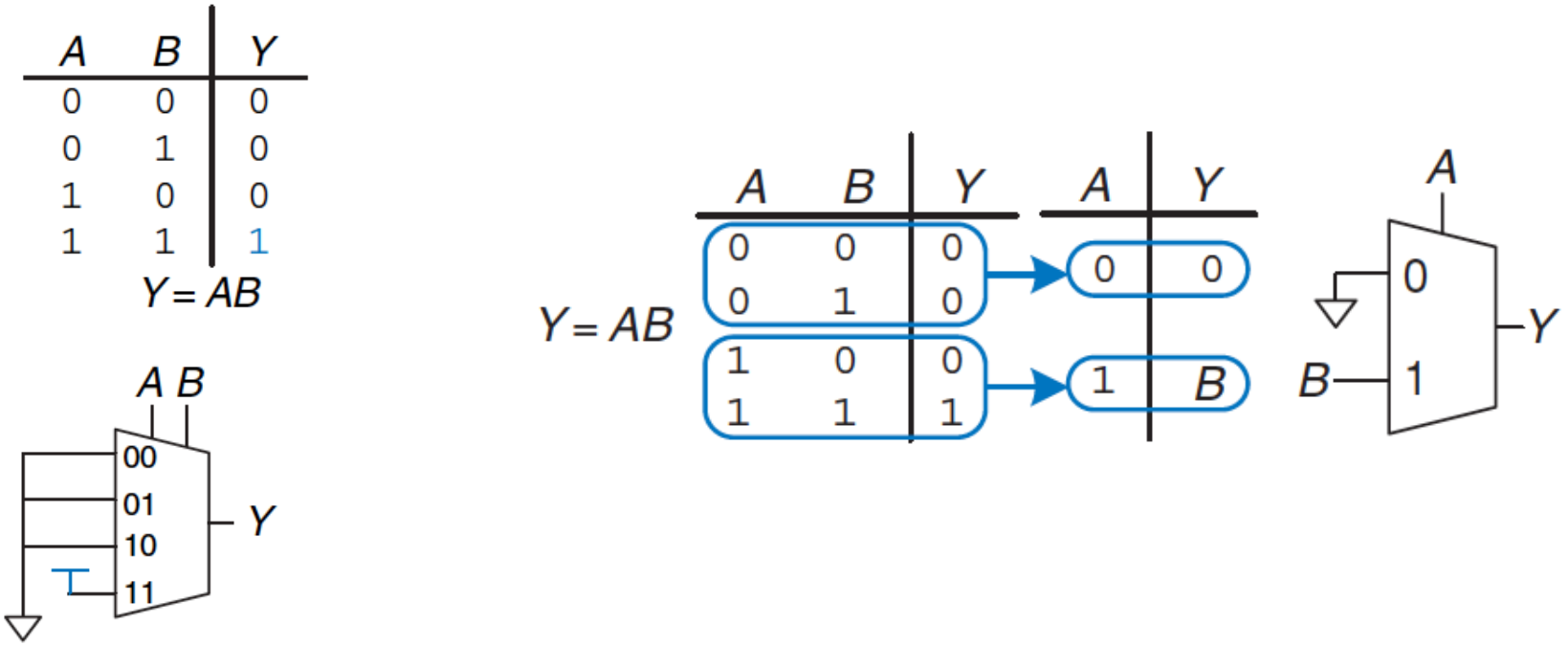
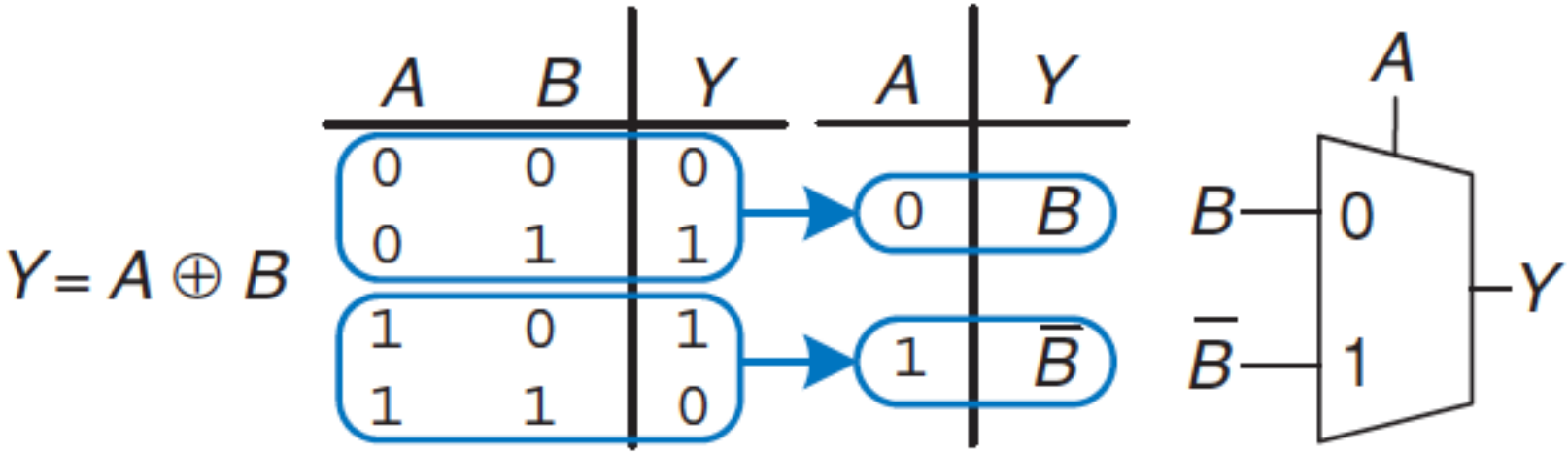


Figure 2.59 4:1 multiplexer implementation of two-input AND function

Aside: Logic Using Multiplexers (II)

- Multiplexers can be used as lookup tables to perform logic functions

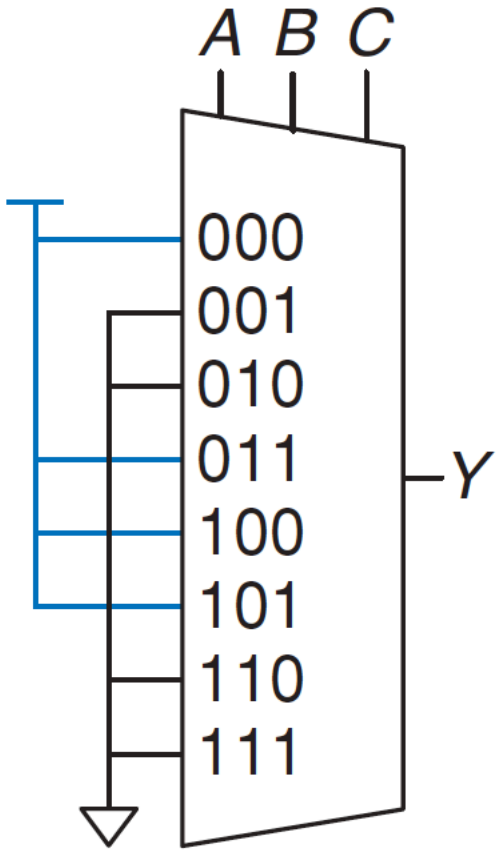


Aside: Logic Using Multiplexers (III)

- Multiplexers can be used as lookup tables to perform logic functions

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$$Y = A\overline{B} + \overline{B}\overline{C} + \overline{A}BC$$



Read H&H Chapter 2.8

Aside: Logic Using Decoders (I)

- Decoders can be combined with OR gates to build logic functions.

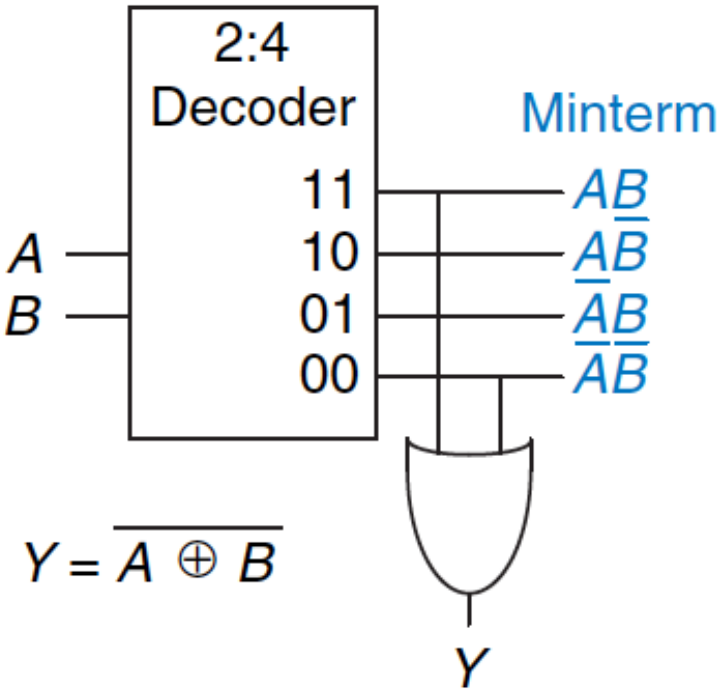
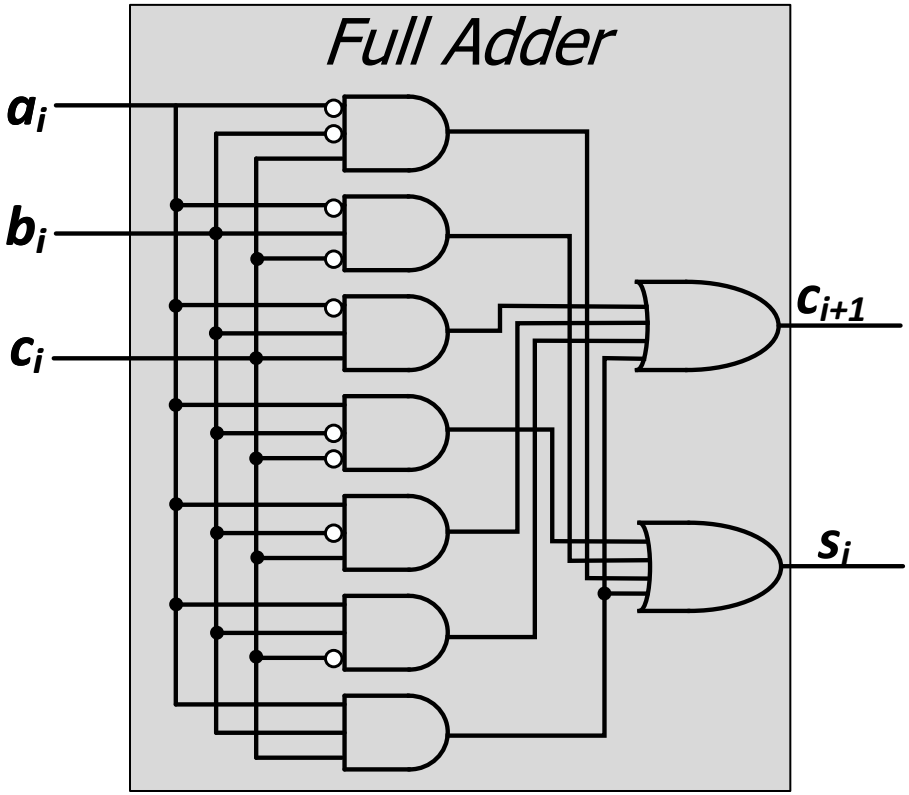


Figure 2.65 Logic function using decoder

Read H&H Chapter 2.8

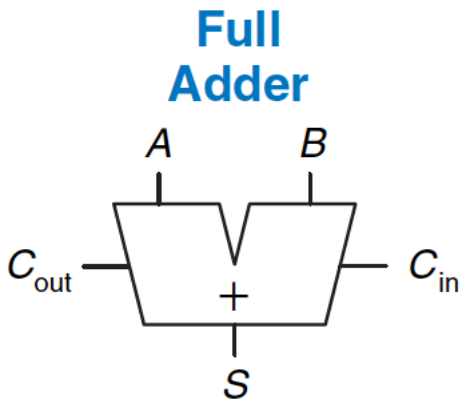
Logic Simplification using Boolean Algebra Rules

Recall: Full Adder in SOP Form Logic



a_i	b_i	$carry_i$	$carry_{i+1}$	S_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Goal: Simplified Full Adder



C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

How do we simplify Boolean logic?

Quick Recap on Logic Simplification

- The original Boolean expression (i.e., logic circuit) may not be optimal

$$F = \sim A(A + B) + (B + AA)(A + \sim B)$$

- Can we reduce a given Boolean expression to an equivalent expression with fewer terms?

$$F = A + B$$

- The goal of logic simplification:
  - Reduce the number of gates/inputs
  - Reduce implementation cost

A basis for what the automated design tools are doing today

Logic Simplification

- Systematic techniques for simplifications
  - amenable to automation

Key Tool: The Uniting Theorem — $F = A\overline{B} + AB$

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

$$F = A\overline{B} + AB = A(\overline{B} + B) = A(1) = A$$

Essence of Simplification:

Find two element subsets of the ON-set where only one variable changes its value. This single varying variable *can be eliminated!*

value is not needed

→ *B is eliminated, A remains*

A	B	G
0	0	1
0	1	0
1	0	1
1	1	0

$$G = \overline{A}\overline{B} + A\overline{B} = (\overline{A} + A)\overline{B} = \overline{B}$$

B's value stays the same within the ON-set rows

A's value changes within the ON-set rows

→ *A is eliminated, B remains*

Logic Simplification:
Karnaugh Maps (K-Maps)

Karnaugh Maps are Fun…

- A pictorial way of minimizing circuits by visualizing opportunities for simplification
- They are for you to **study on your own**…

- See Backup Slides
- Read H&H Section 2.7
- Watch videos of Lectures 5 and 6 from 2019 Digitech course:
  - https://youtu.be/0ks0PeaOUjE?list=PL5Q2soXY2Zi8J58xLKBNFQFHRO3GrXxA9&t=4570
  - https://youtu.be/ozs18ARNG6s?list=PL5Q2soXY2Zi8J58xLKBNFQFHRO3GrXxA9&t=220

Sequential Logic Circuits and Design

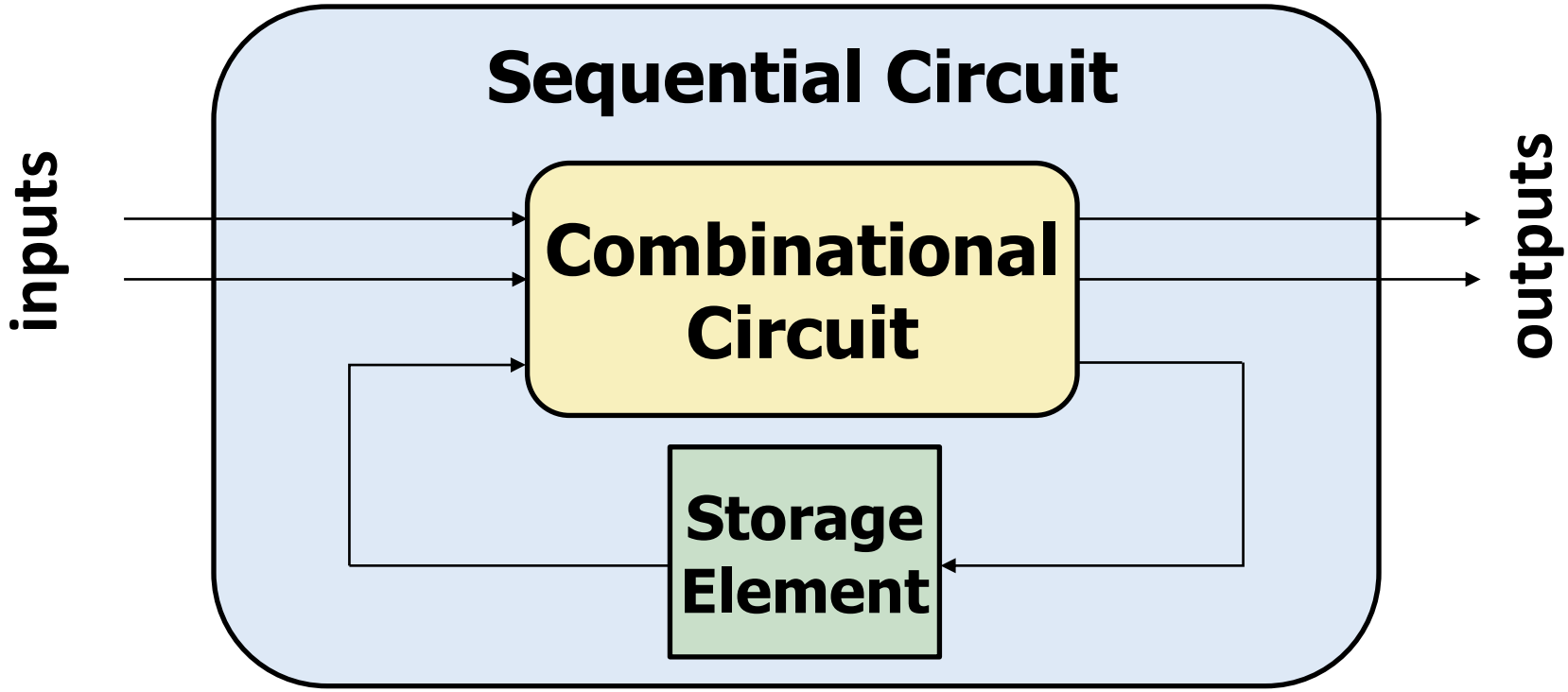
What We Will Learn Today

- Circuits that can store information
  - Cross-coupled inverter
  - R-S Latch
  - Gated D Latch
  - D Flip-Flop
  - Register
- Finite State Machines (FSM)
  - Moore Machine
  - Mealy Machine
- Verilog implementations of sequential circuits (next week)

Circuits that Can Store Information

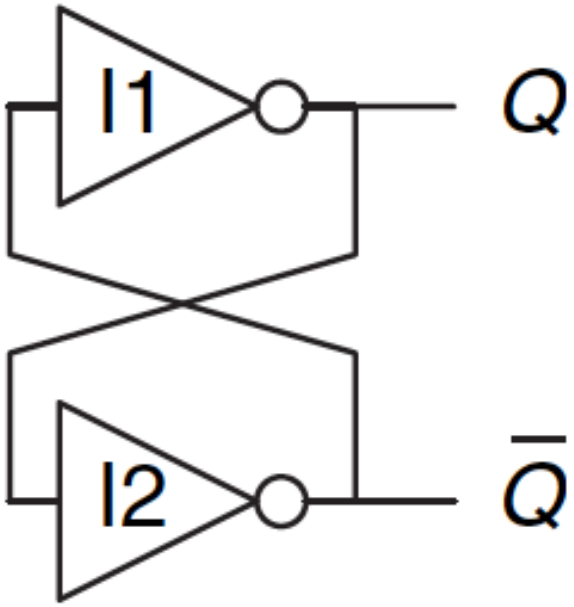
Introduction

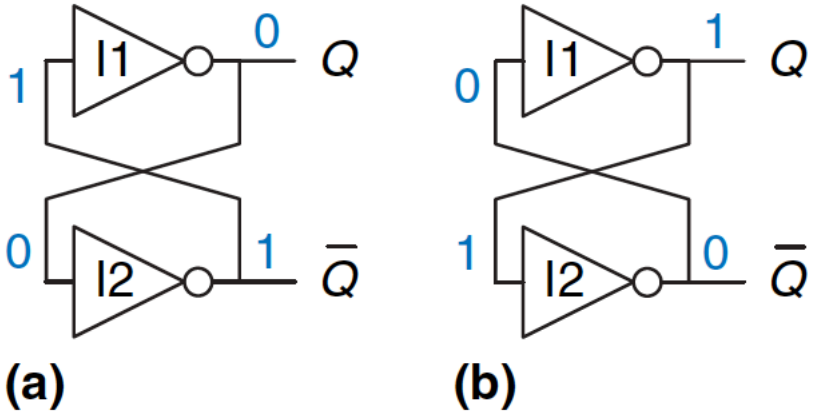
- Combinational circuit output depends **only** on current input
- We want circuits that produce output depending on **current** and **past** input values – circuits with **memory**
- How can we design a circuit that **stores information**?



Capturing Data

Basic Element: Cross-Coupled Inverters



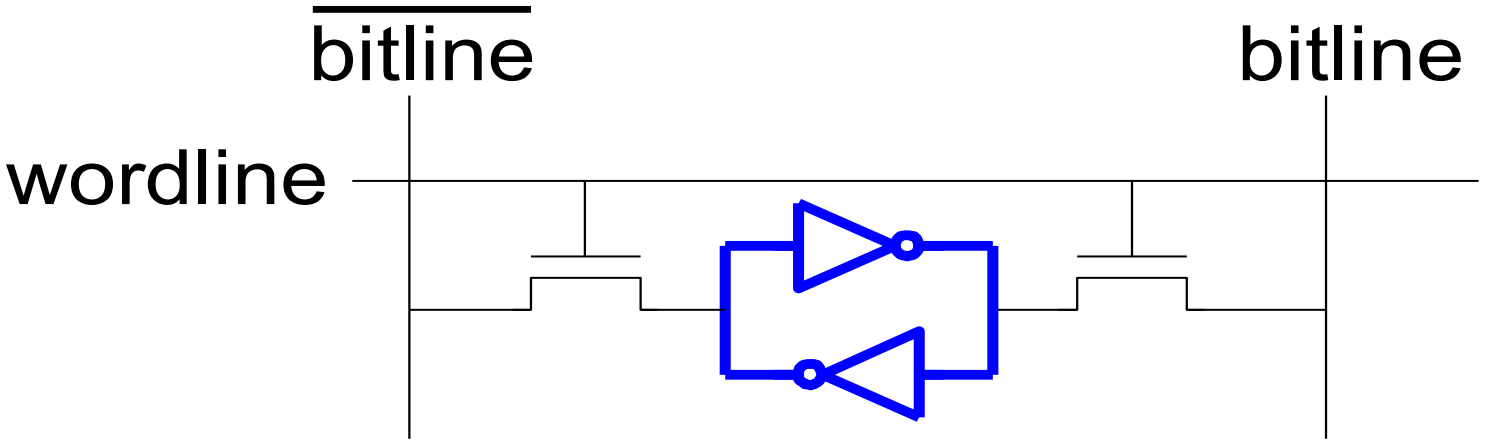


- Has two stable states: Q=1 or Q=0.
- Has a third possible "metastable" state with both outputs oscillating between 0 and 1 (we will see this later)
- Not useful without a *control mechanism* for setting Q

Image source: Harris and Harris, Digital Design and Computer Architecture, 2nd Ed., p.110.

More Realistic Storage Elements

- **Have a control mechanism for setting Q**
  - We will see the R-S latch soon
  - Let's look at an SRAM (static random access memory) cell first



SRAM cell

- We will get back to SRAM (and DRAM) later

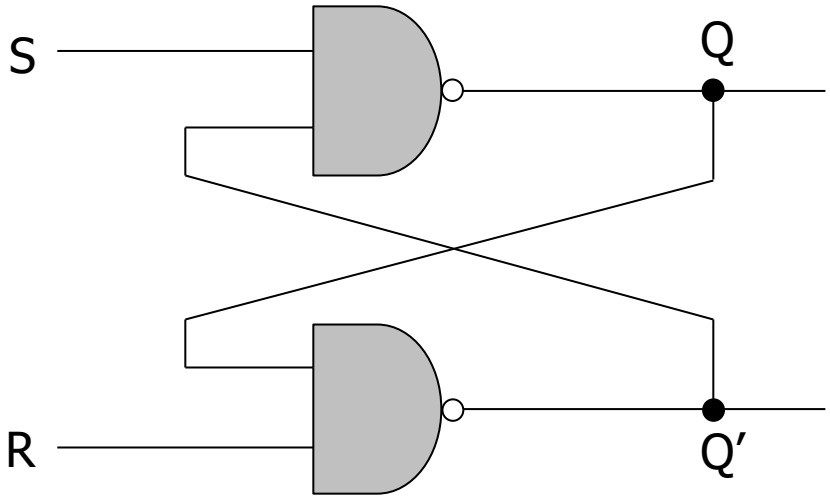
The Big Picture: Storage Elements

- Latches and Flip-Flops
  - Very fast, parallel access
  - Very expensive (one bit costs tens of transistors)
- Static RAM (SRAM)
  - Relatively fast, only one data word at a time
  - Expensive (one bit costs 6+ transistors)
- Dynamic RAM (DRAM)
  - Slower, one data word at a time, reading destroys content (refresh), needs special process for manufacturing
  - Cheap (one bit costs only one transistor plus one capacitor)
- Other storage technology (flash memory, hard disk, tape)
  - Much slower, access takes a long time, non-volatile
  - Very cheap

Basic Storage Element: The R-S Latch

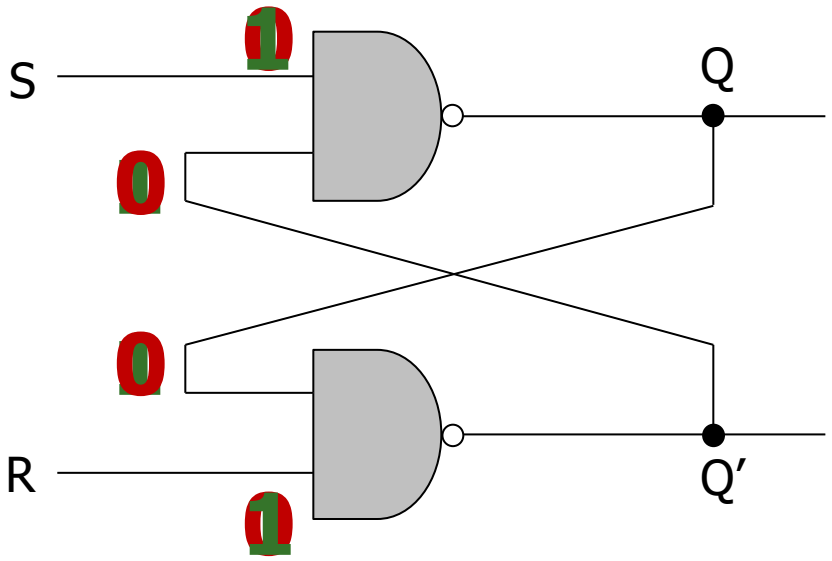
The R-S (Reset-Set) Latch

- Cross-coupled **NAND gates**
  - Data is stored at **Q** (inverse at **Q'**)
  - **S** and **R** are control inputs
    - In *quiescent* (*idle*) *state,* **both S and R are held at 1**
    - **S (set):** drive **S** to 0 (keeping **R** at 1) to change **Q** to 1
    - **R (reset):** drive **R** to 0 (keeping **S** at 1) to change **Q** to 0
- **S** and **R** should never **both** be 0 at the same time



Input		Output
R	S	Q
1	1	Q_{prev}
1	0	1
0	1	0
0	0	Forbidden

Why not R=S=0?



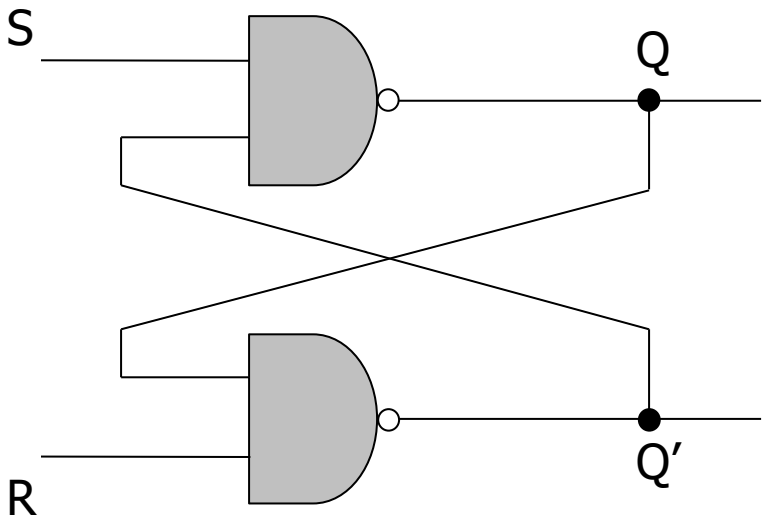
Input		Output
R	S	Q
1	1	Q_{prev}
1	0	1
0	1	0
0	0	Forbidden

1. If **R=S=0, Q** and **Q'** will both settle to 1, which **breaks** our invariant that **Q** = !**Q'**
2. If **S** and **R** transition back to 1 at the same time, **Q** and **Q'** begin to oscillate between 1 and 0 because their final values depend on each other (**metastability**)
   - This eventually settles depending on **variation in the circuits** (more **metastability** to come in **Lecture 8**)

The Gated D Latch

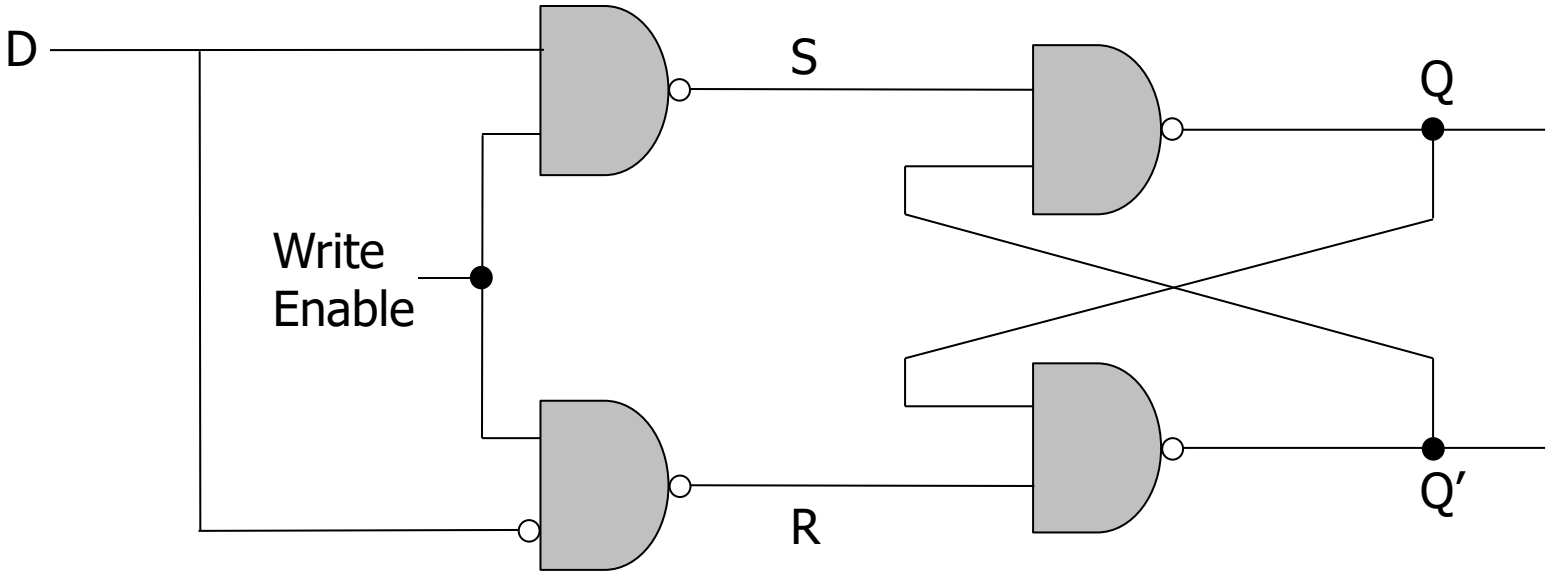
The Gated D Latch

- How do we **guarantee** correct operation of an R-S Latch?



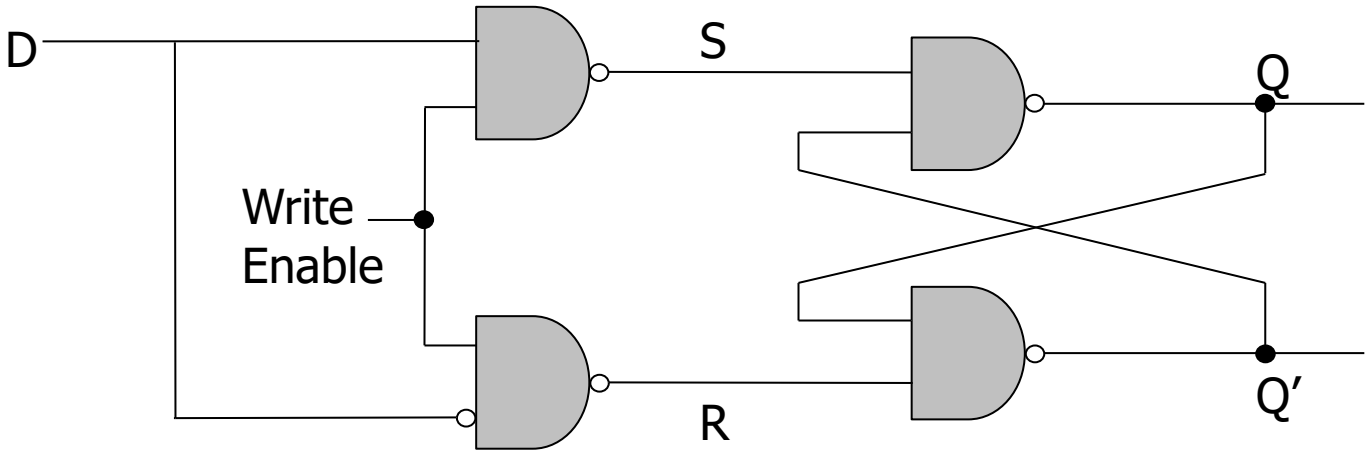
The Gated D Latch

- How do we **guarantee** correct operation of an R-S Latch?
  - Add two more NAND gates!



  - **Q** takes the value of **D**, when **write enable (WE)** is set to 1
  - **S** and **R** can never be 0 at the same time!

The Gated D Latch



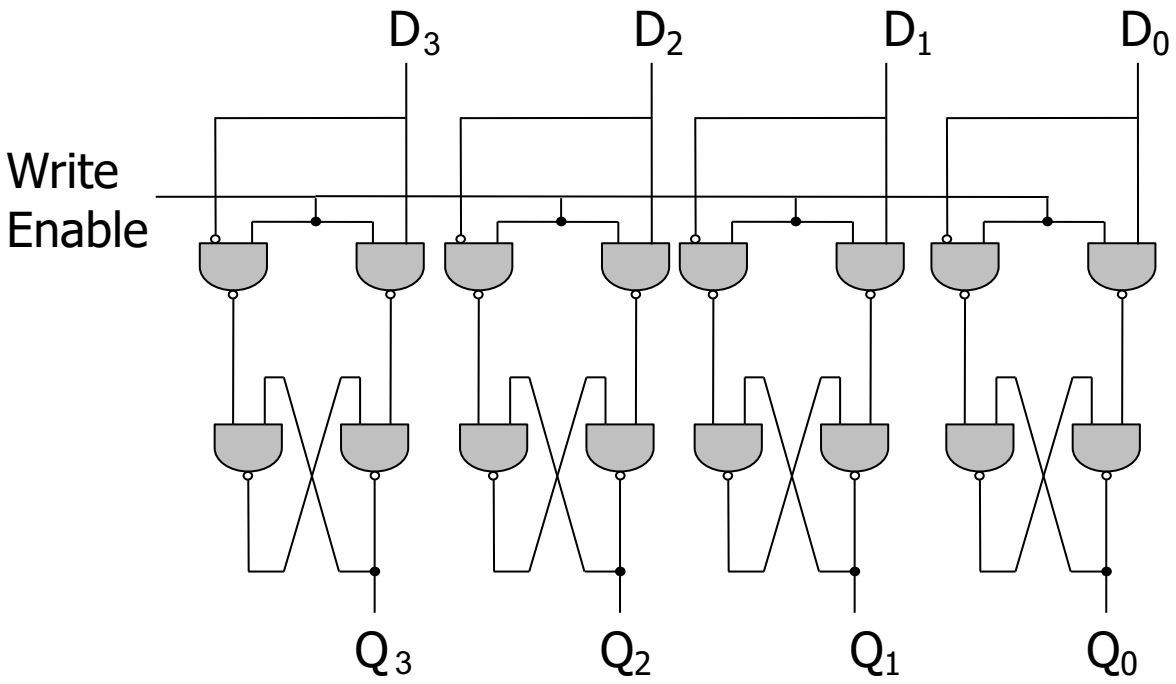
Input		Output
WE	D	Q
0	0	Q_{prev}
0	1	Q_{prev}
1	0	0
1	1	1

The Register

The Register

How can we use D latches to store **more** data?

- Use **more** D latches!
- A single WE signal for all latches for simultaneous writes



Here we have a **register,** or a structure that stores more than one bit and can be read from and written to

This **register** holds 4 bits, and its data is referenced as Q[3:0]

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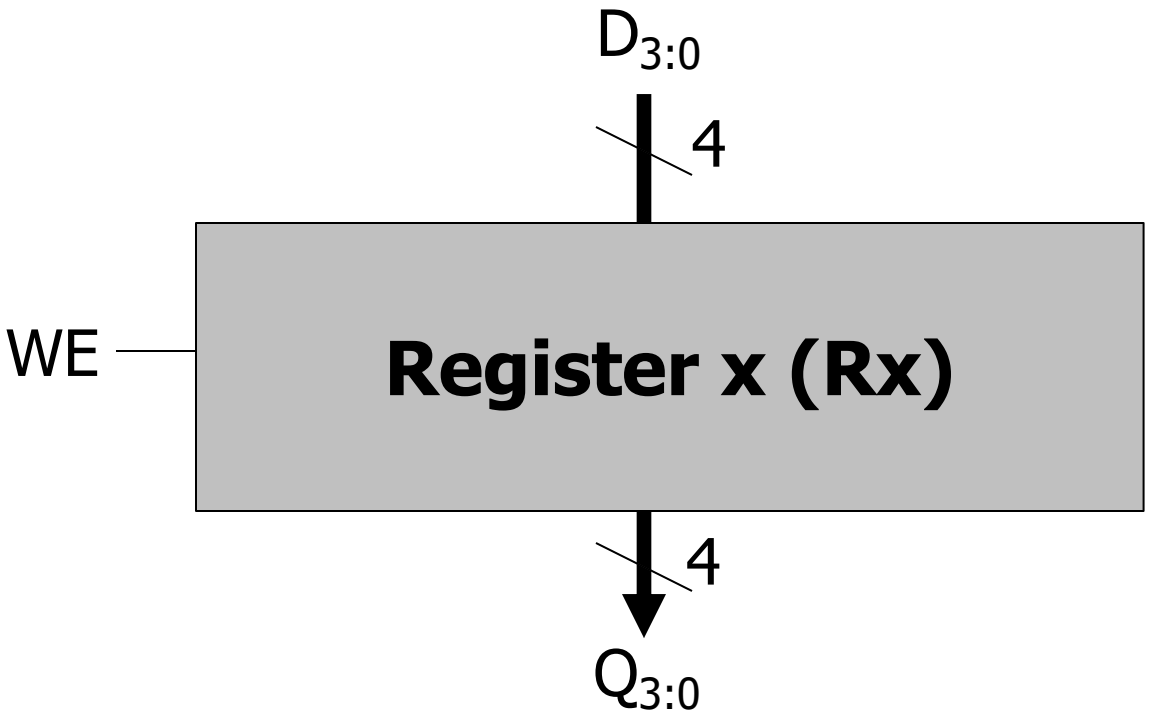

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Memory

Memory

- **Memory** is comprised of locations that can be written to or read from. An example memory array with 4 locations:

Addr(00): 0100 1001	**Addr**(01): 0100 1011
Addr(10): 0010 0010	**Addr**(11): 1100 1001

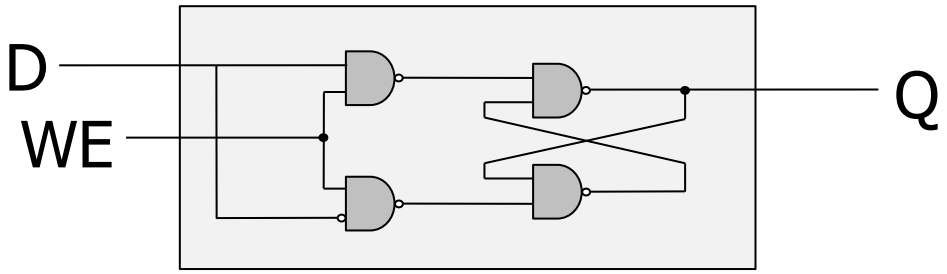
- Every unique location in memory is indexed with a unique **address.** 4 locations require 2 address bits (log[#locations]).
- **Addressability:** the number of bits of information stored in each location. This example: addressability is 8 bits.
- The entire set of unique locations in memory is referred to as the **address space.**
- Typical memory is **MUCH** larger (billions of locations)

Addressing Memory

Let's implement a simple memory array with:

- 3-bit addressability **&** address space size of 2 (total of 6 bits)

1 Bit



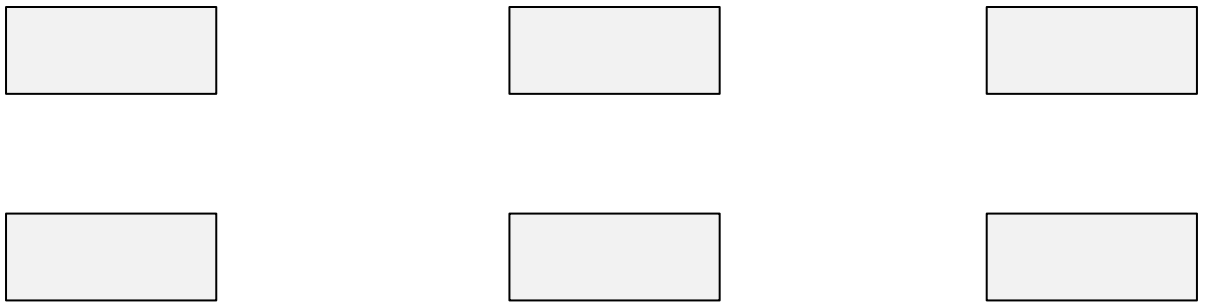
6-Bit Memory Array

Addr(0)	Bit_2	Bit_1	Bit_0
Addr(1)	Bit_2	Bit_1	Bit_0

Reading from Memory

How can we select the address to read?

- Because there are 2 addresses, address size is log(2)=1 bit



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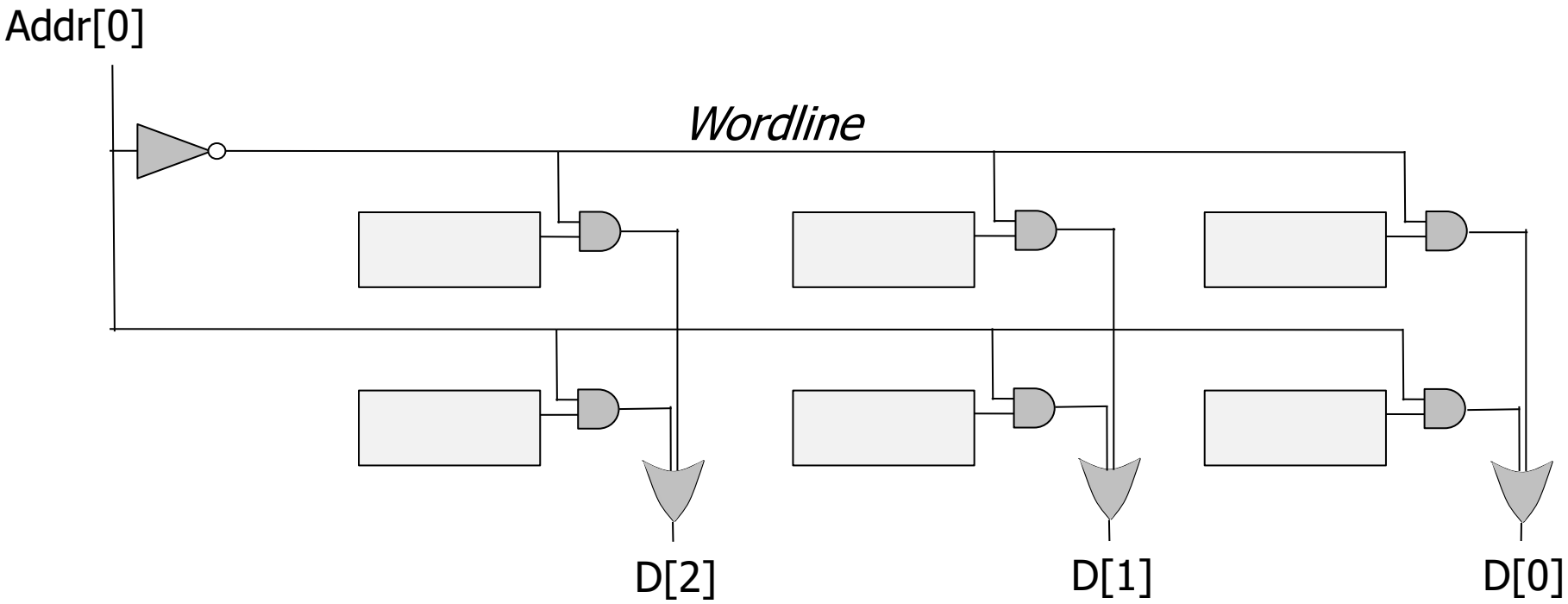

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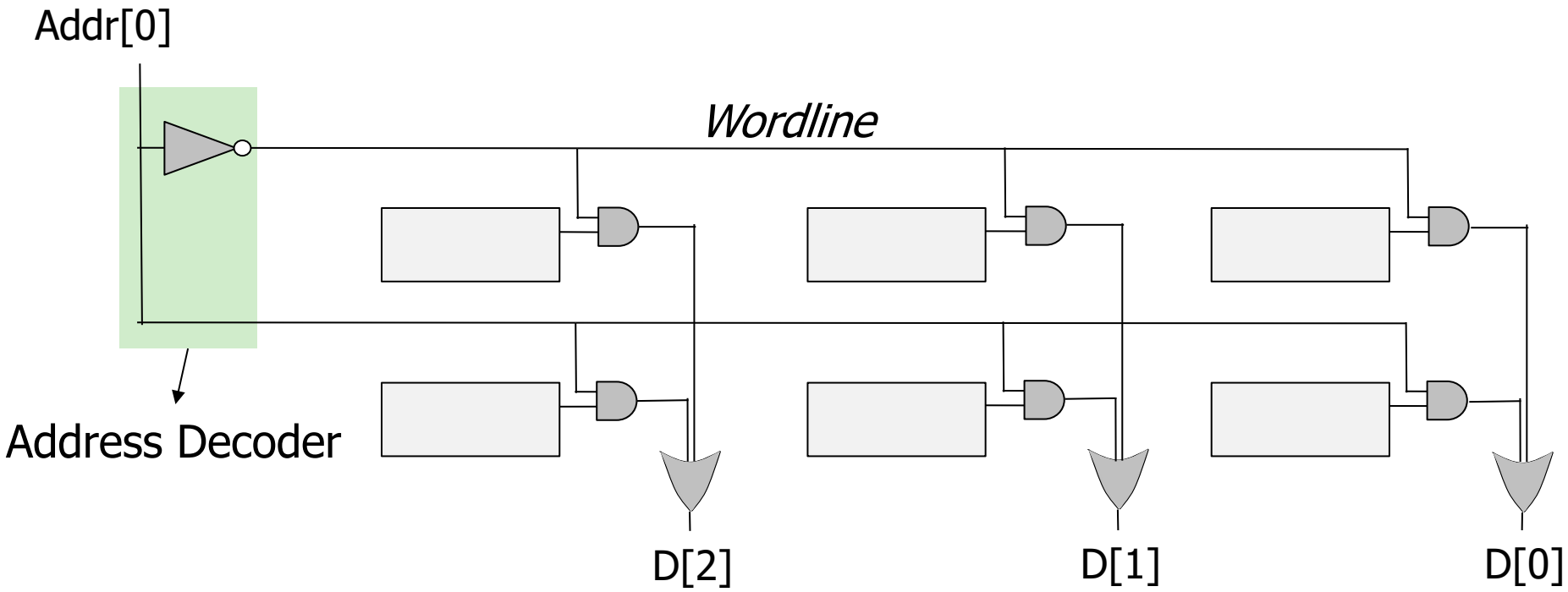

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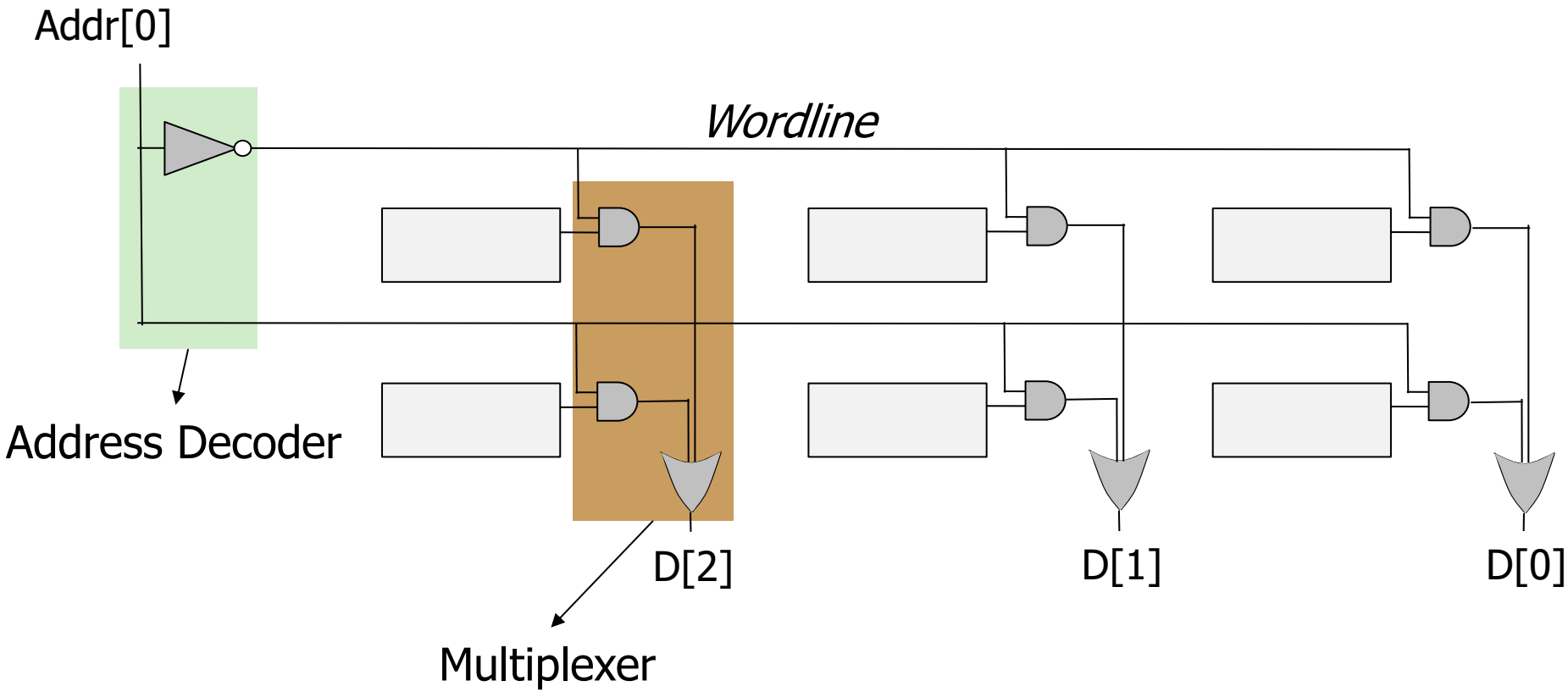
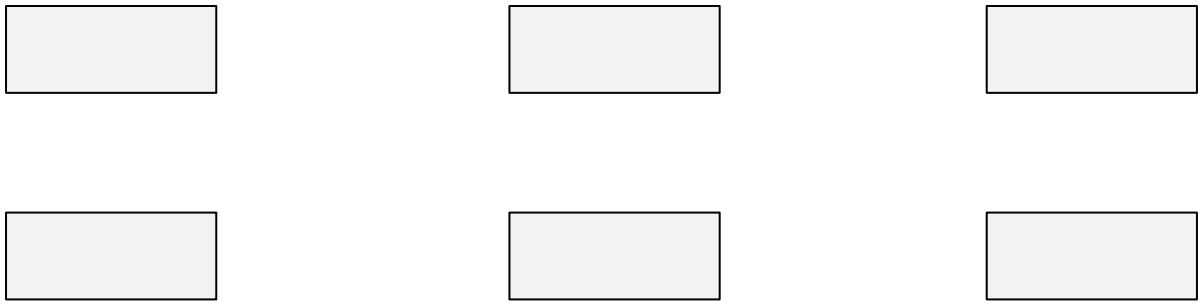


Writing to Memory

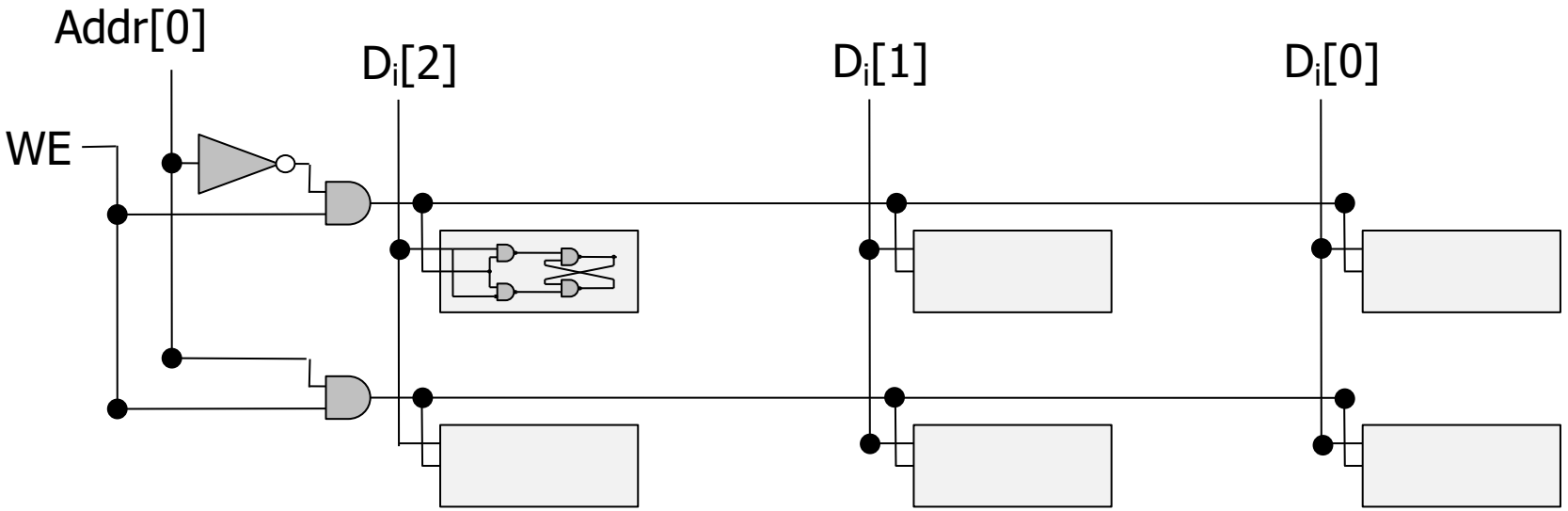
How can we select an address and write to it?



Writing to Memory

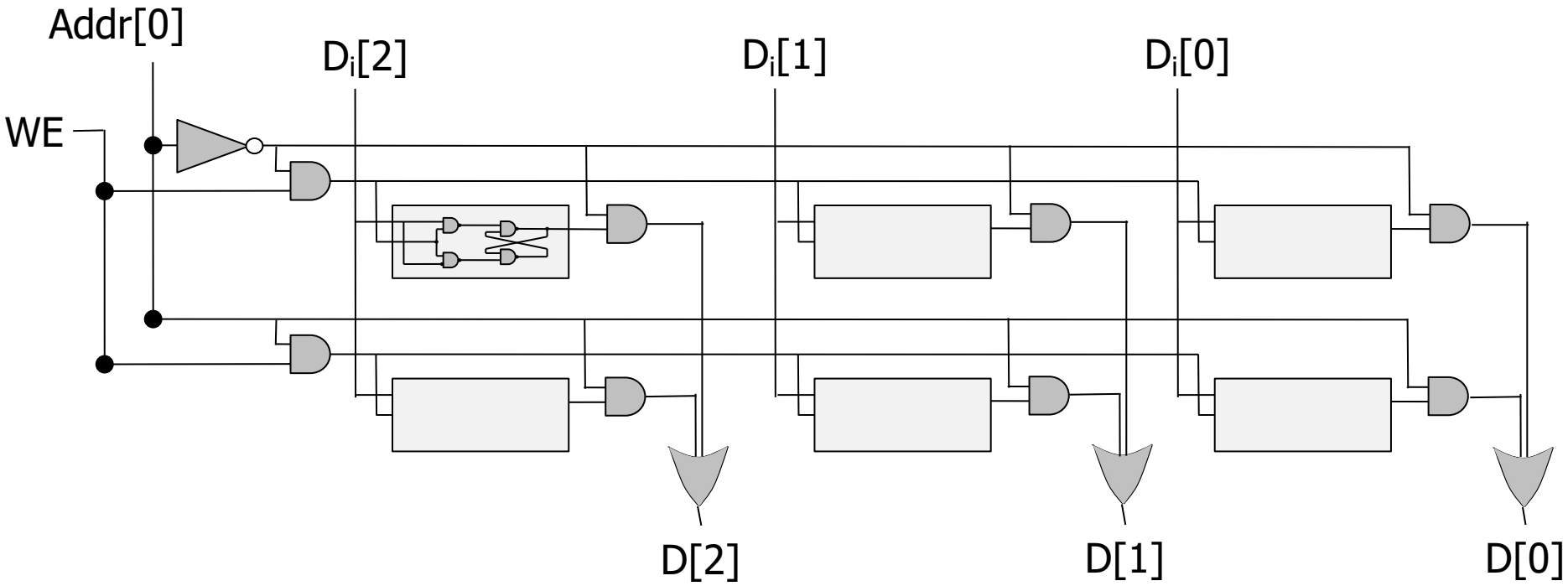
How can we select an address and write to it?

- Input is indicated with D_i

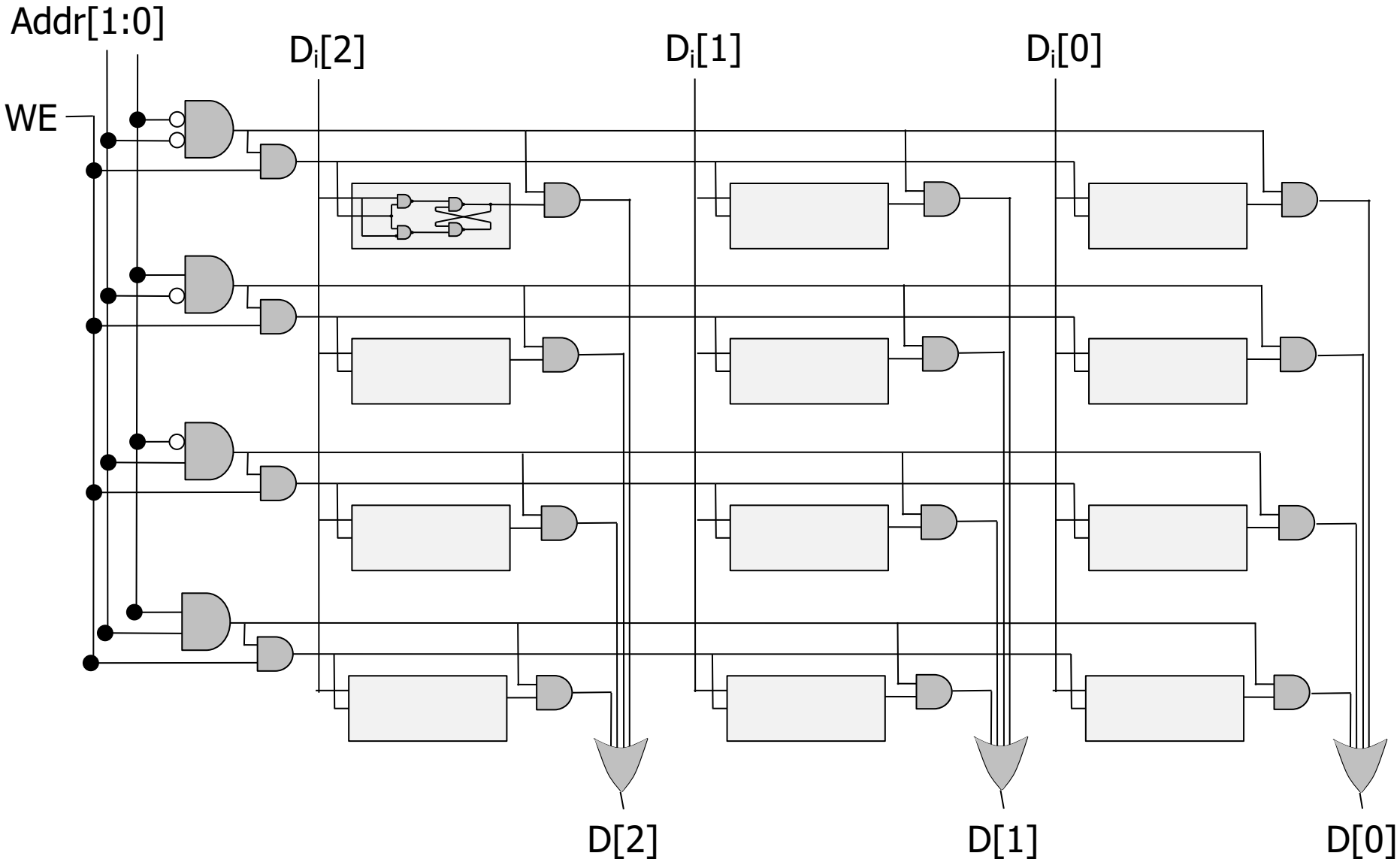


Putting it all Together

Let's enable reading and writing to a memory array



A Bigger Memory Array



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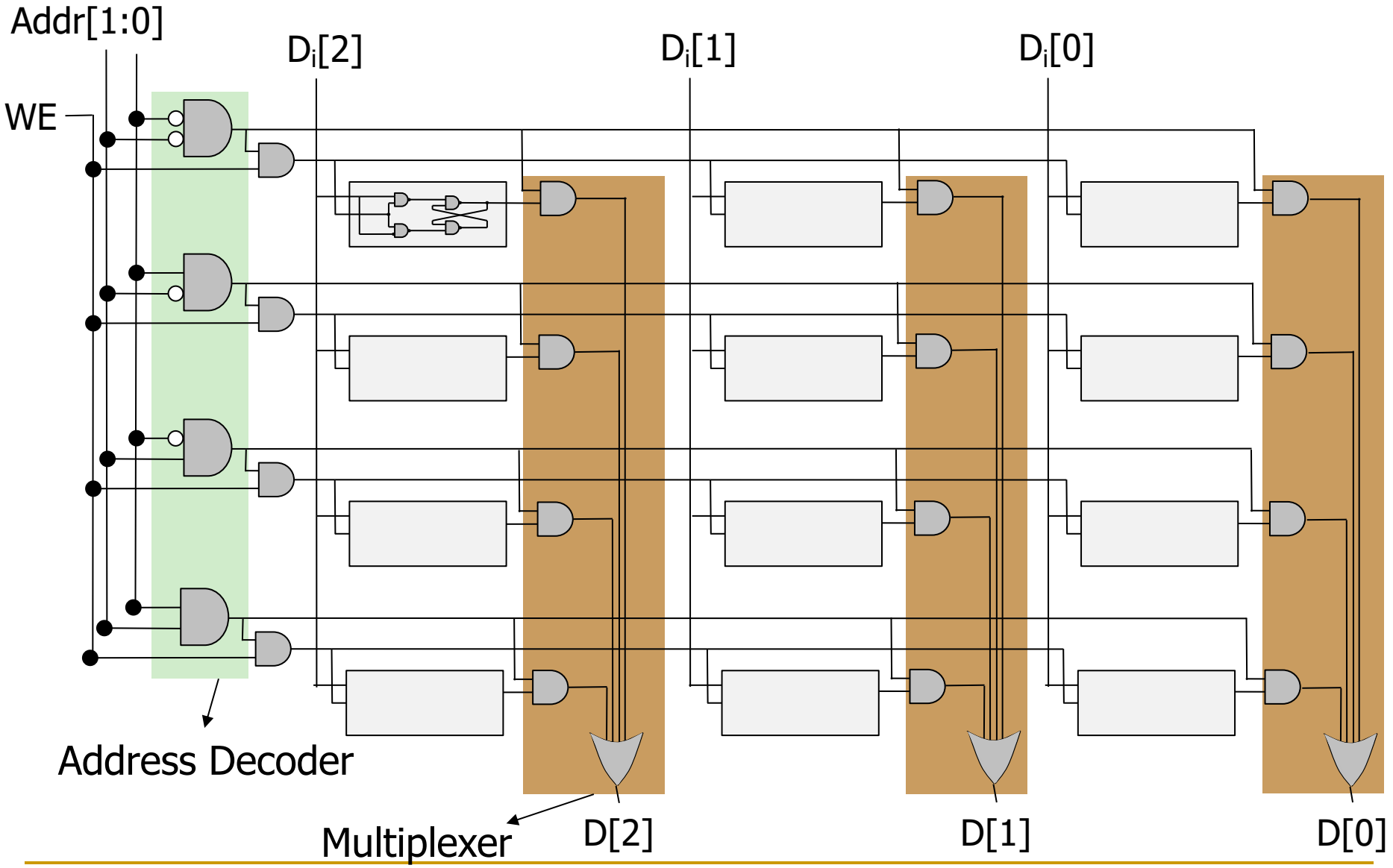


Sequential Logic Circuits

Sequential Logic Circuits

- We have looked at designs of circuit elements that can **store information**
- Now, we will use these elements to build circuits that **remember** past inputs

Combinational
Only depends on current inputs

Sequential
Opens depending on past inputs

https://www.easykeys.com/228_ESP_Combination_Lock.aspx
https://www.fosmon.com/product/tsa-approved-lock-4-dial-combo

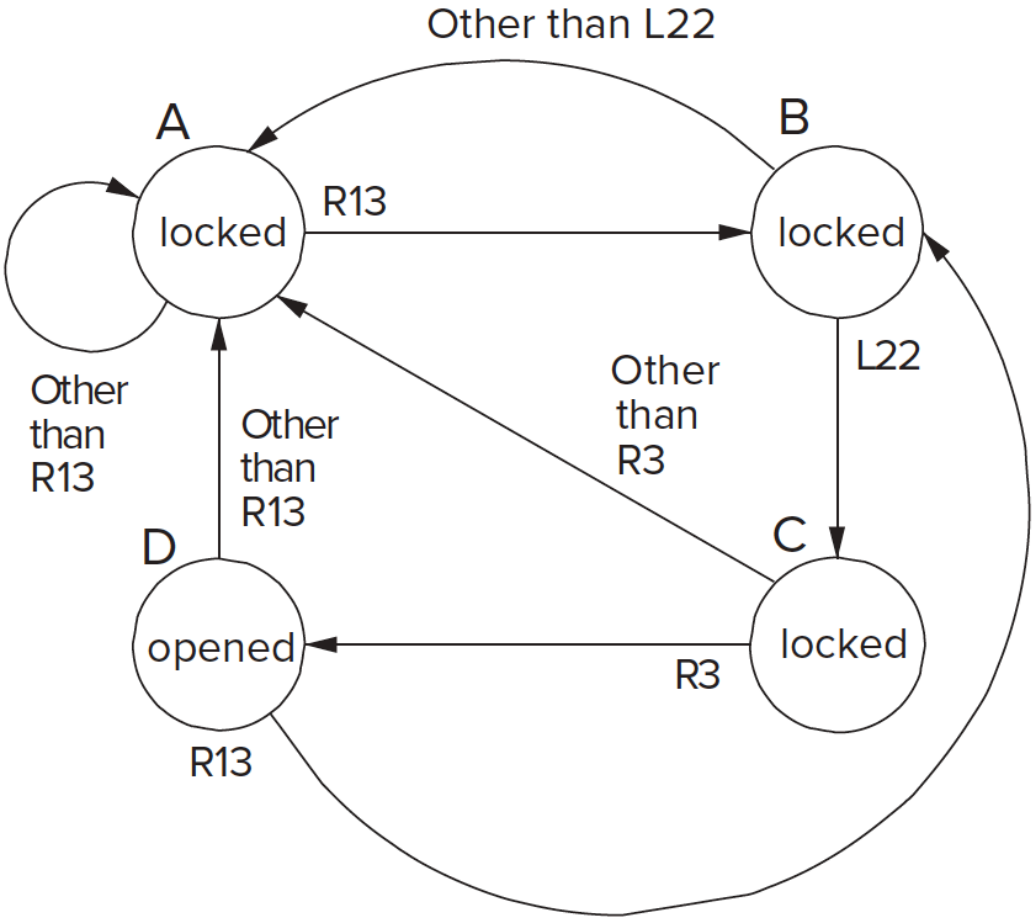
State

- In order for this lock to work, it has to keep track (**remember**) of the past events!
- If passcode is **R13-L22-R3**, sequence of **states** to unlock:
  A. The lock is not open (locked), and no relevant operations have been performed
  B. Locked but user has completed R13
  C. Locked but user has completed R13-L22
  D. Unlocked: user has completed R13-L22-R3

- The **state** of a system is a snapshot of all relevant elements of the system at the moment of the snapshot
  - To open the lock, **states A-D must be completed in order**
  - If anything else happens (e.g., L5), lock **returns** to state A

State Diagram of Our Sequential Lock

- Completely describes the operation of the sequential lock

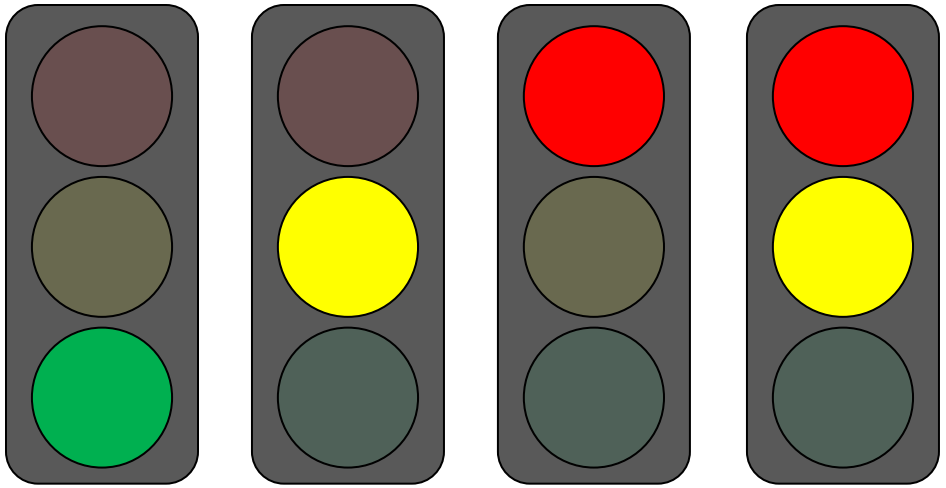


- We will understand "state diagrams" fully later today

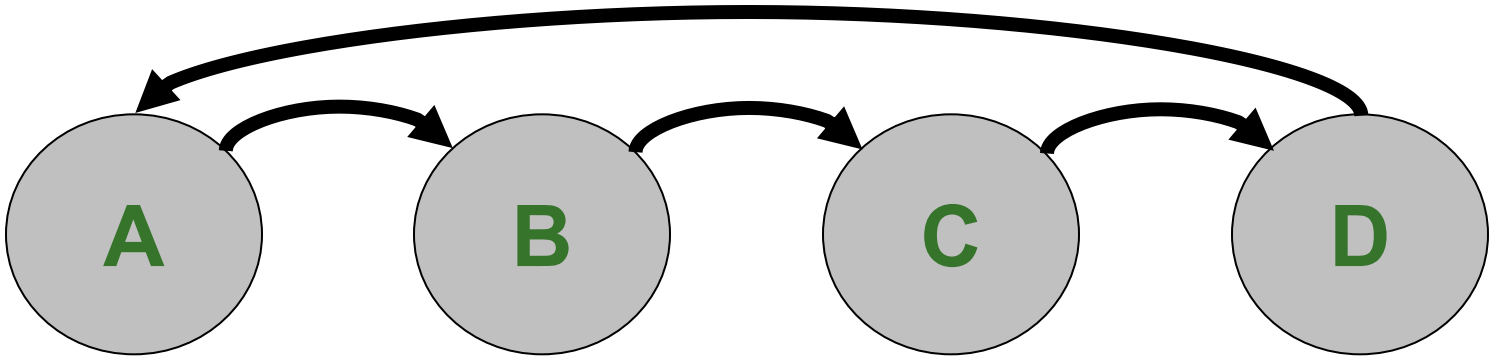
Image source: Patt and Patel, "Introduction to Computing Systems", 2nd ed., page 76.

Another Simple Example of State

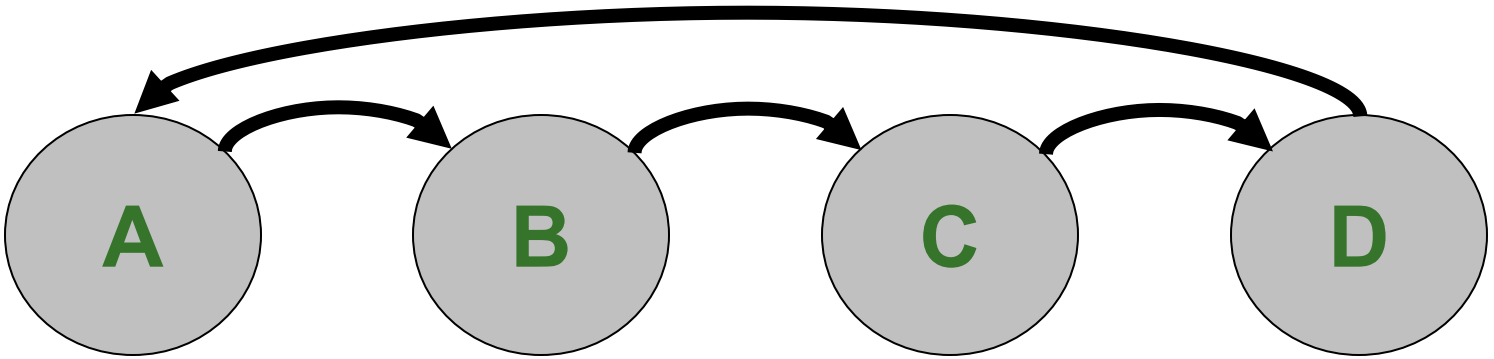
- A standard Swiss traffic light has **4 states**
  A. Green
  B. Yellow
  C. Red
  D. Red and Yellow



- The sequence of these states are always as follows



Changing State: The Notion of Clock (I)



- When should the light change from one state to another?
- We need a **clock** to dictate when to change state
  - Clock signal alternates between 0 & 1

- At the start of a clock cycle ( ), system state changes
  - During a clock cycle, the state stays constant
  - In this traffic light example, we are assuming the traffic light stays in each state an equal amount of time

Changing State: The Notion of Clock (II)

- **Clock** is a general mechanism that triggers transition from one state to another in a sequential circuit
- Clock synchronizes state changes across many sequential circuit elements
- Combinational logic evaluates for the length of the clock cycle
- Clock cycle should be chosen to accommodate maximum combinational circuit delay
  - More on this later, when we discuss timing

Finite State Machines

Finite State Machines

- What is a **Finite State Machine** (FSM)?
  - **A discrete-time model** of a stateful system
  - Each state represents a snapshot of the system at a given time

- An FSM pictorially shows
  1. the set of all possible **states** that a system can be in
  2. how the system transitions from one state to another

- An FSM can model
  - A traffic light, an elevator, fan speed, a microprocessor, etc.

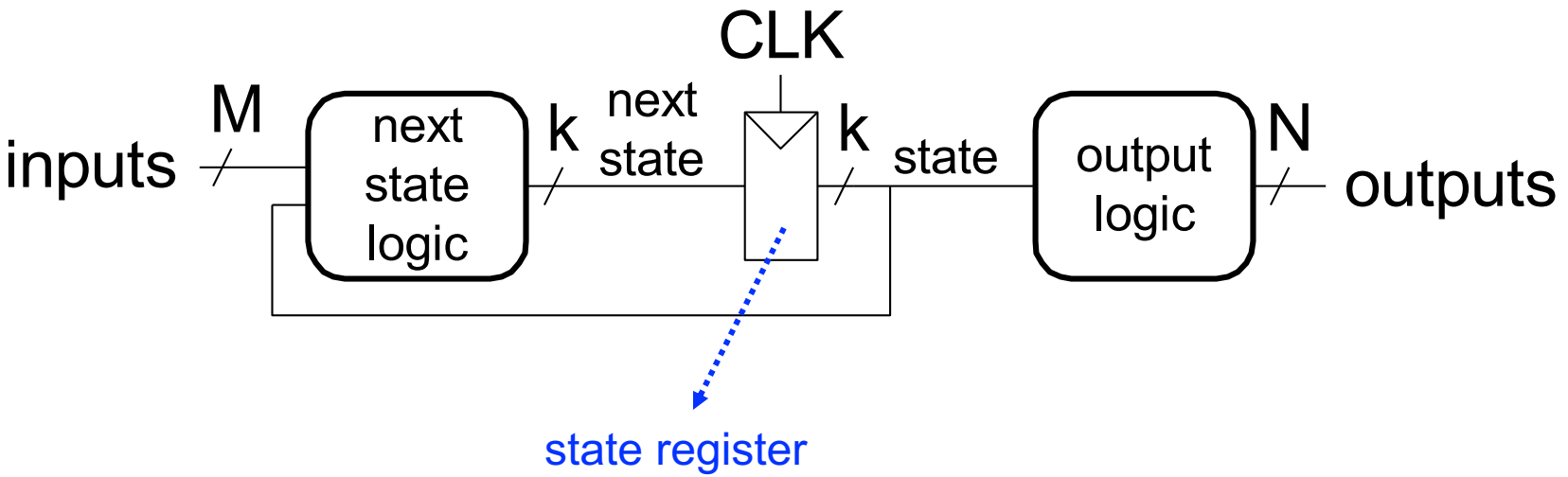
- **An FSM enables us to pictorially think of a stateful system using simple diagrams**

Finite State Machines (FSMs) Consist of:

- **Five elements:**
  1. A **finite** number of states
     - ***State***: snapshot of all relevant elements of the system at the time of the snapshot
  2. A **finite** number of external inputs
  3. A **finite** number of external outputs
  4. An explicit specification of all state transitions
     - How to get from one state to another
  5. An explicit specification of what determines each external output value

Finite State Machines (FSMs)

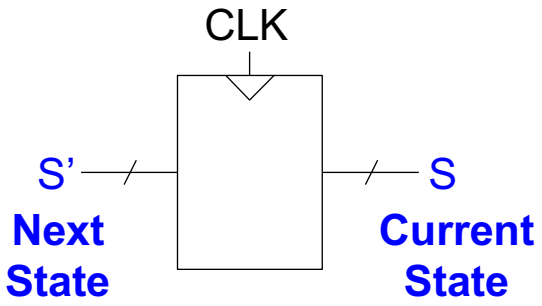
- Each FSM consists of three separate parts:
  - next state logic
  - state register
  - output logic



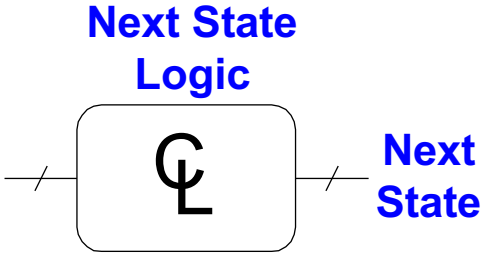
At the beginning of the clock cycle, next state is latched into the state register

Finite State Machines (FSMs) Consist of:

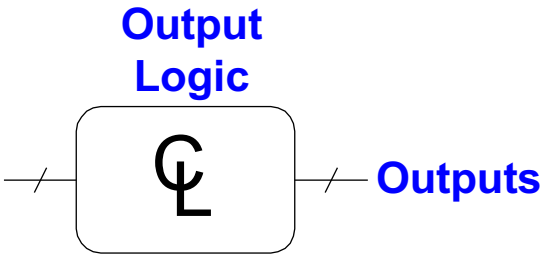
- **Sequential circuits**
  - State register(s)
    - Store the current state and
    - Load the next state at the clock edge



- **Combinational Circuits**
  - Next state logic
    - Determines what the next state will be



  - Output logic
    - Generates the outputs



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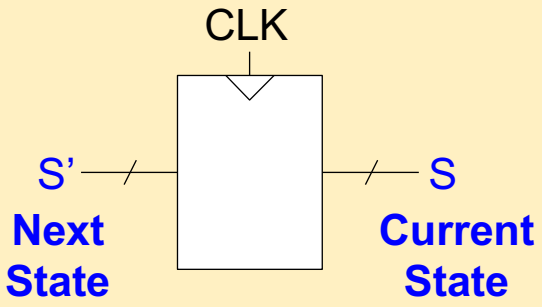

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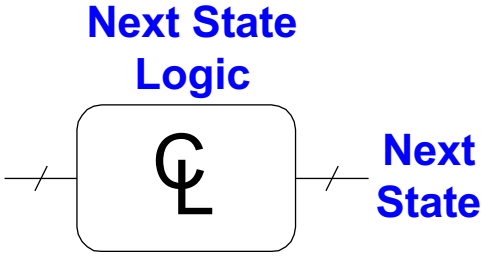

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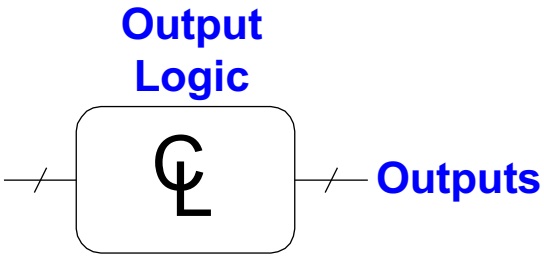
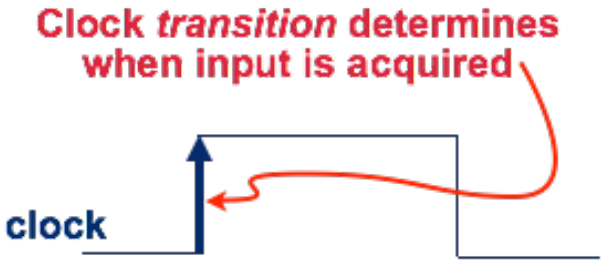


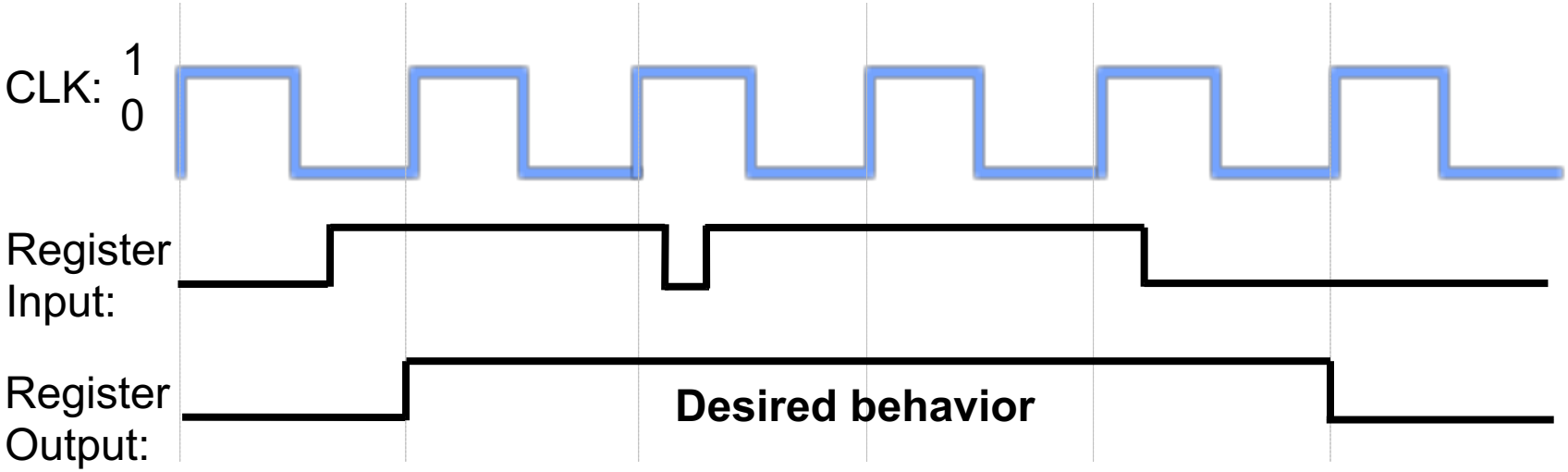
State Register Implementation

- How can we implement a **state register**? Two properties:

  1. We need to store data at the **beginning** of every clock cycle

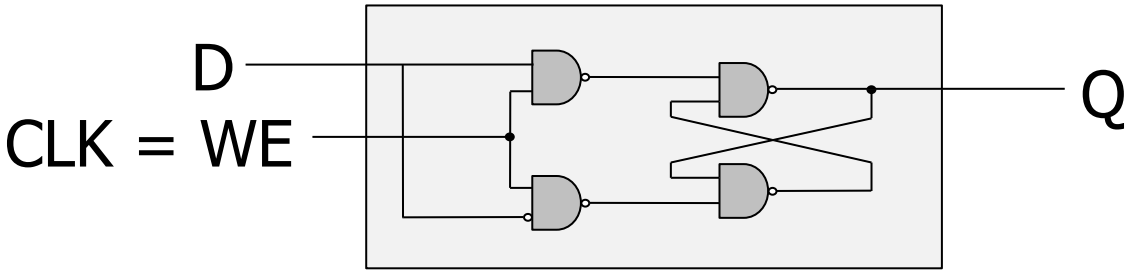


  2. The data must be **available** during the entire clock cycle

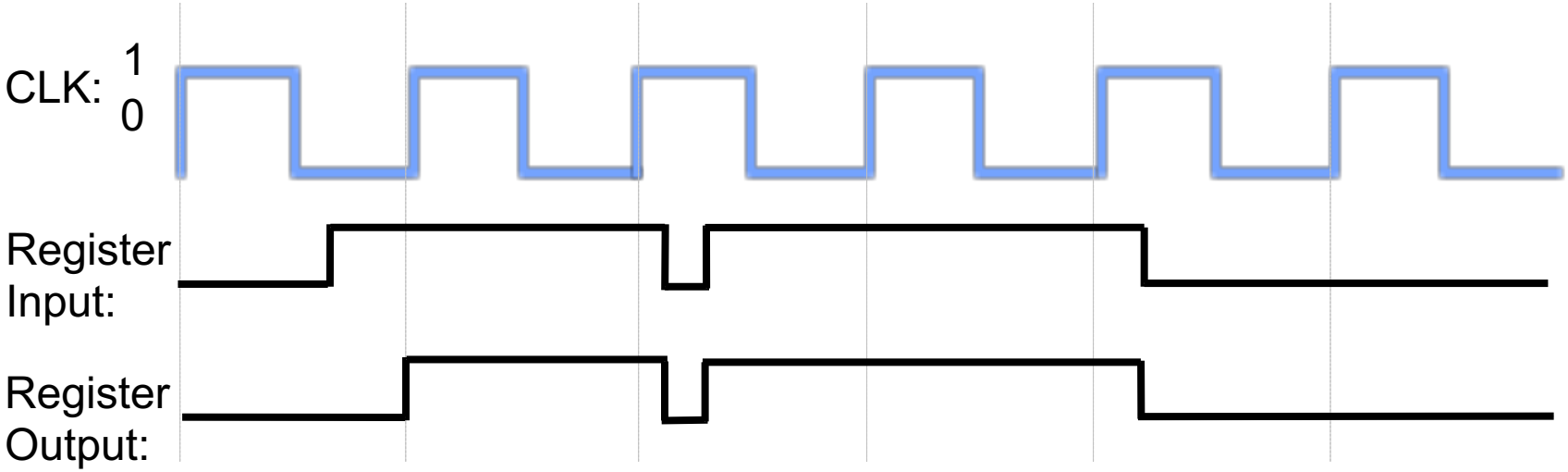


The Problem with Latches

Recall the Gated D Latch



- Currently, we **cannot** simply wire a clock to WE of a latch
  - **Whenever the clock is high,** the latch propagates **D** to **Q**
  - **The latch is transparent**



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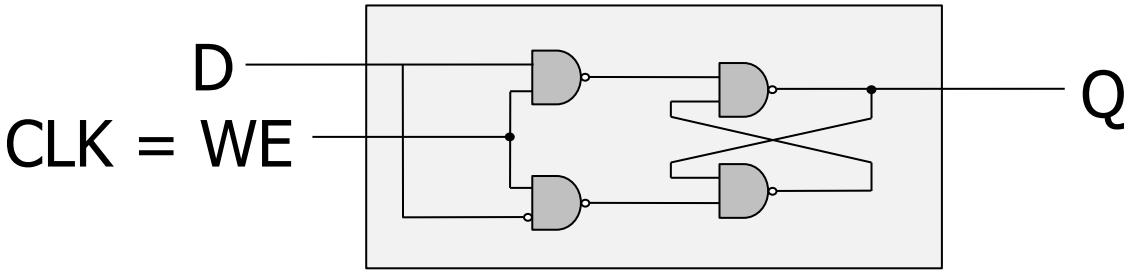

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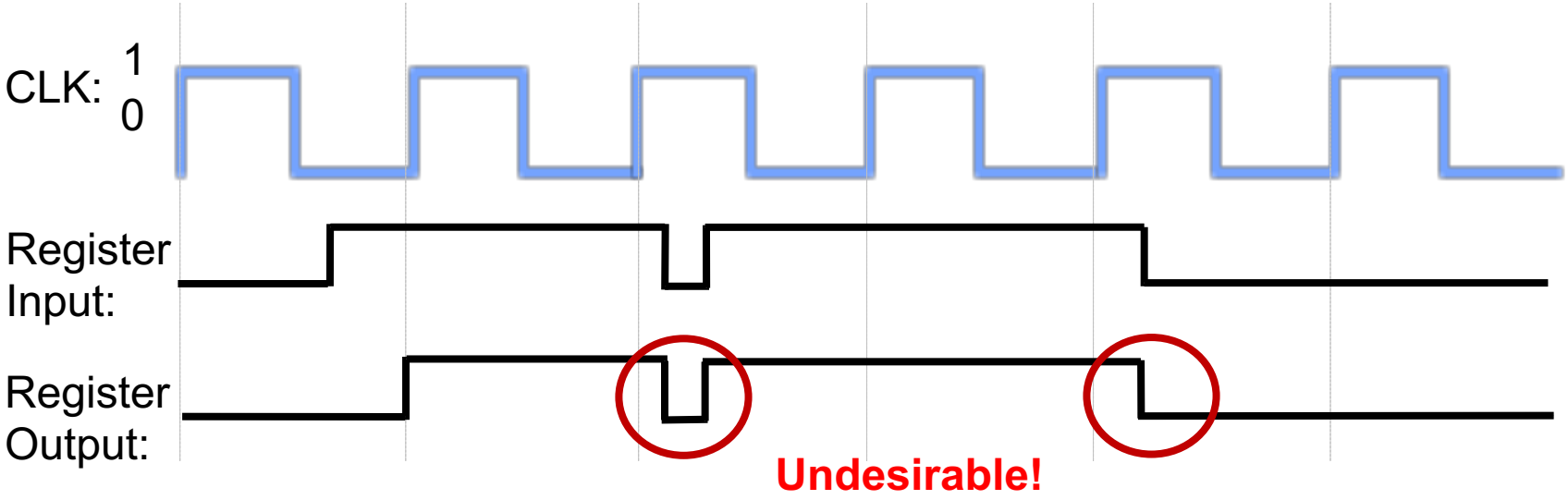

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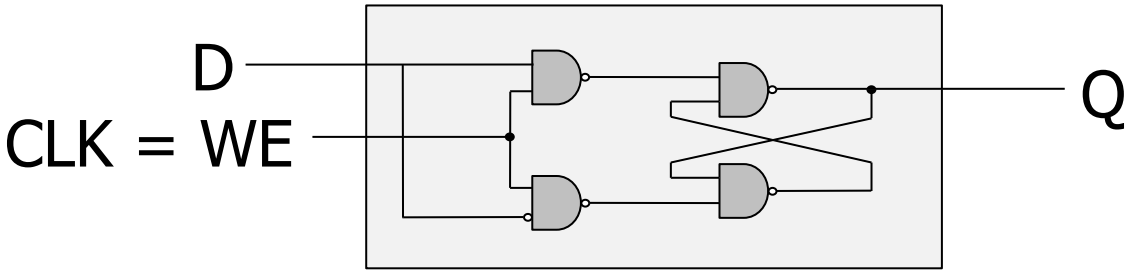


How can we change the latch, so that

1) D (input) is **observable** at **Q** (output) **only** at the **beginning of next** clock cycle?

2) Q is **available for the full clock cycle**

The Need for a New Storage Element

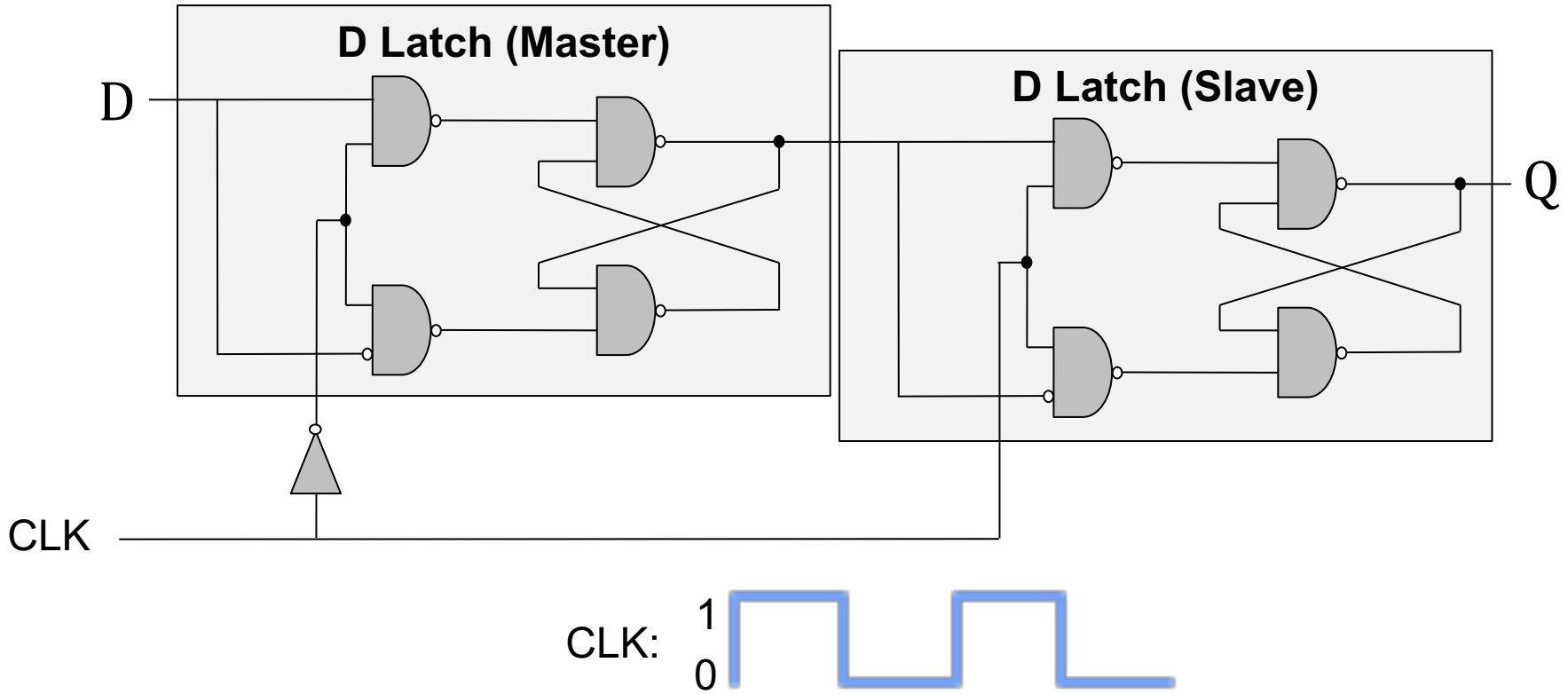
- To design viable FSMs
- We need storage elements that allow us
  - to read the **current state** throughout the **current clock cycle**

  AND

  - not write the **next state** values into the storage elements until the beginning of the **next clock cycle**.

The D Flip-Flop

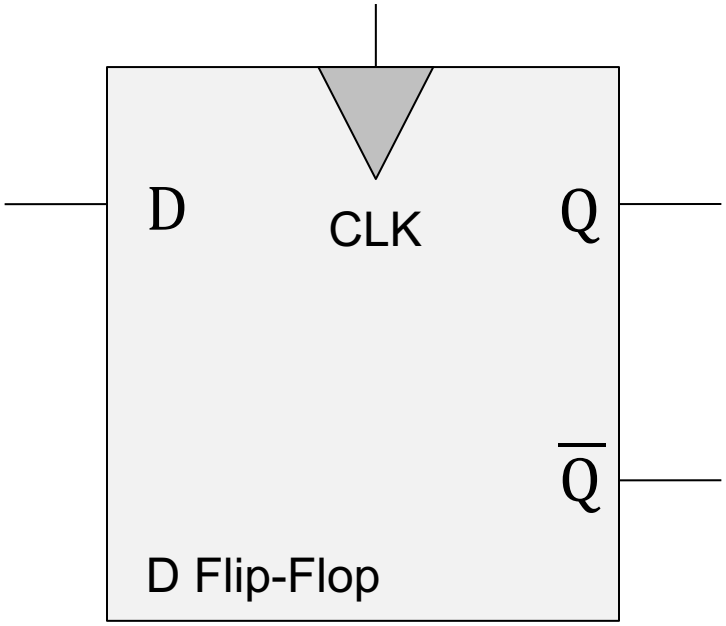
- 1) state change on clock edge, 2) data available for full cycle



- When the clock is low, master propagates **D** to the input of slave (Q unchanged)
- Only when the clock is high, slave latches **D (Q stores D)**
  - At the rising edge of clock (clock going from 0->1), Q gets assigned D

The D Flip-Flop

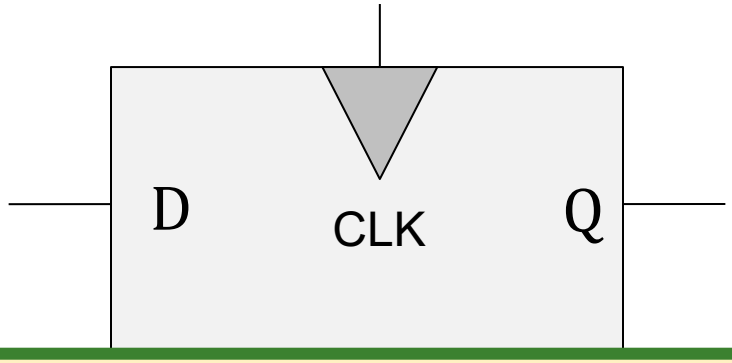
- 1) state change on clock edge, 2) data available for full cycle



- At the rising edge of clock (clock going from 0->1), **Q** gets assigned **D**
- At all other times, Q is unchanged

The D Flip-Flop

- How do we implement this?



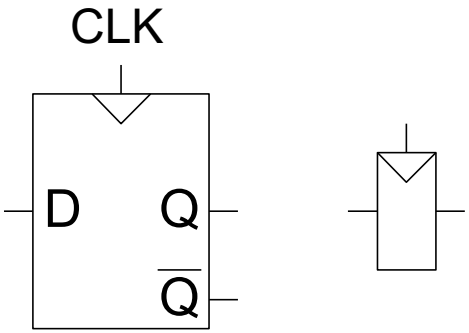
We can use these **Flip-Flops** to implement the state register!

- At the rising edge of clock (clock going from 0->1), **Q** gets assigned **D**
- At all other times, Q is unchanged

Rising-Clock-Edge Triggered Flip-Flop

- **Two inputs**: CLK, D

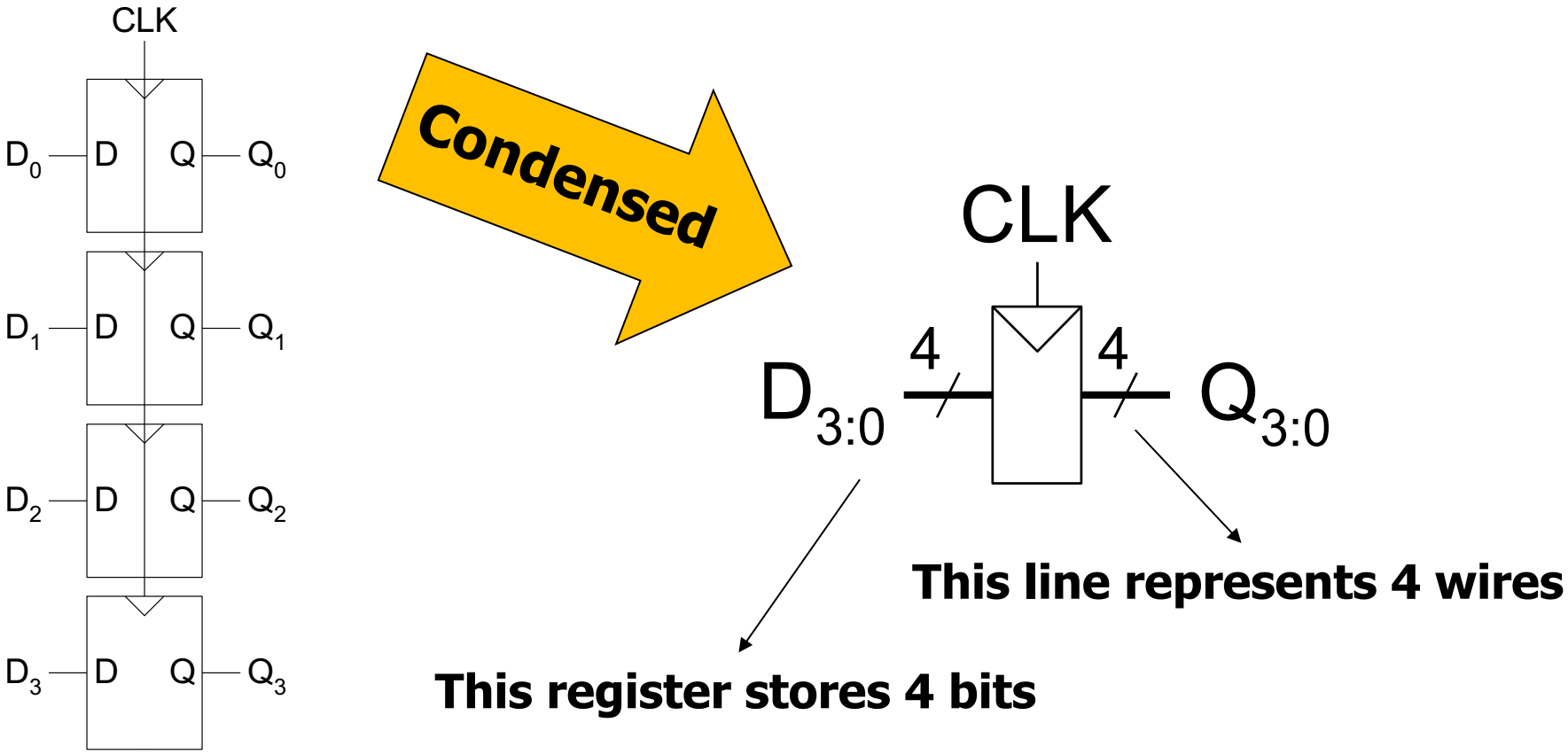
- **Function**
  - The flip-flop "samples" **D** on the rising edge of CLK (**positive edge**)
  - When CLK rises from 0 to 1, **D** passes through to **Q**
  - Otherwise, **Q** holds its previous value
  - **Q** changes **only** on the rising edge of CLK



- A flip-flop is called an **edge-triggered state element** because it captures data on the clock edge
  - A latch is a level-triggered state element

Register

- Multiple parallel flip-flops, each of which storing 1 bit



A 4-Bit D-Flip-Flop-Based Register (Internally)

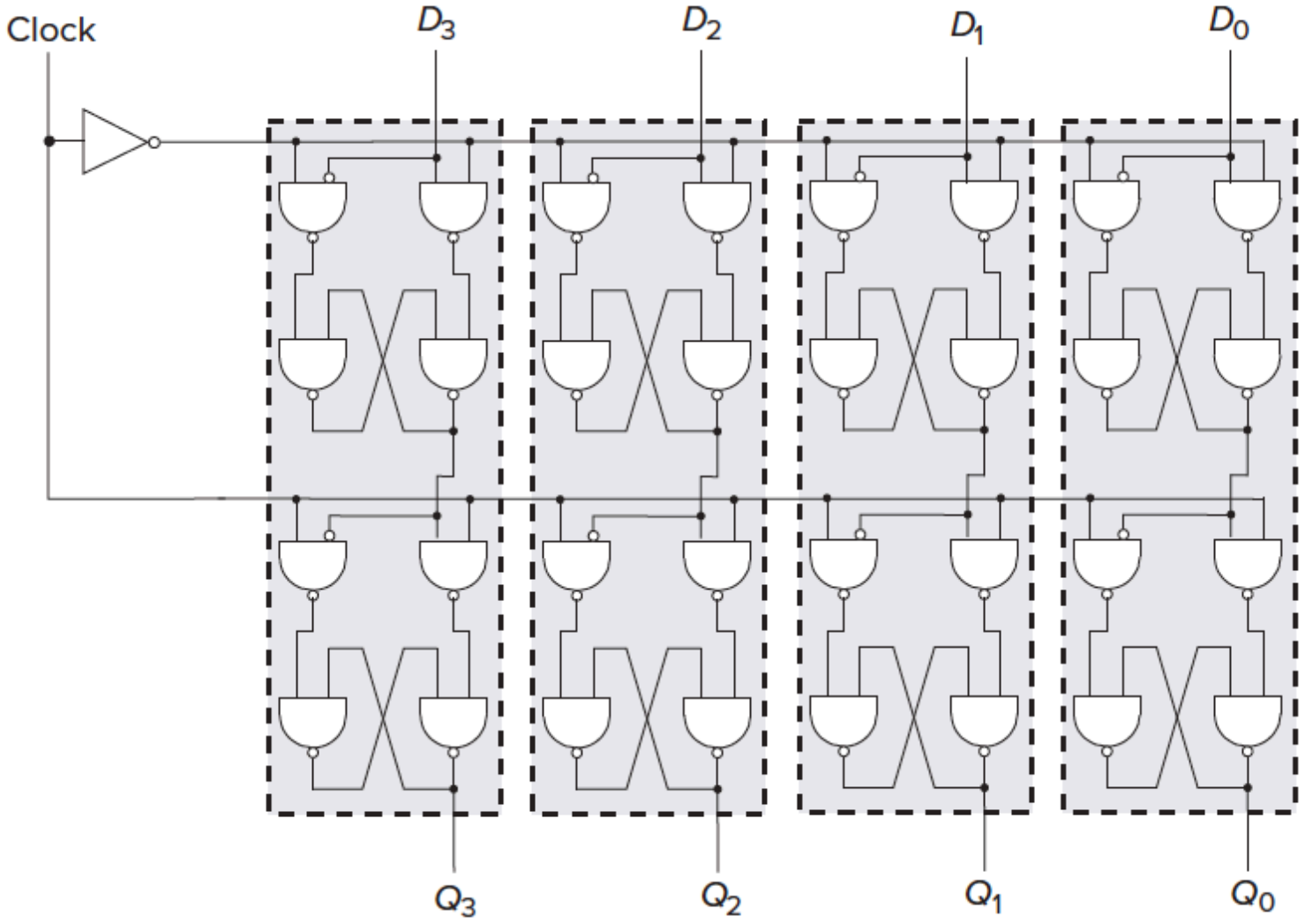
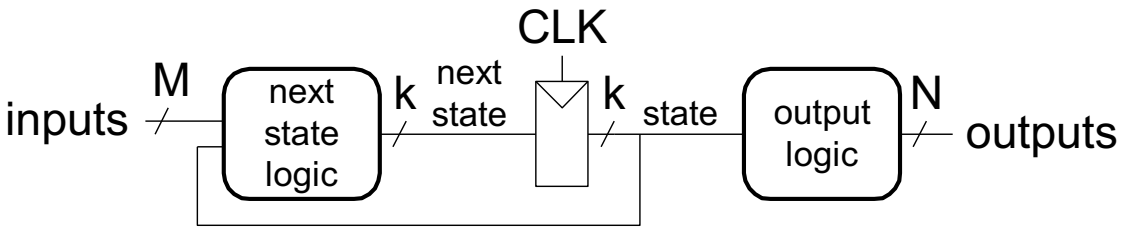


Image source: Patt and Patel, "Introduction to Computing Systems", 3rd ed., tentative page 95.

Finite State Machines (FSMs)

- Next state is determined by the current state and the inputs
- Two types of finite state machines differ in the **output logic**:
  - **Moore FSM**: outputs depend only on the current state

Moore FSM



Finite State Machines (FSMs)

- Next state is determined by the current state and the inputs
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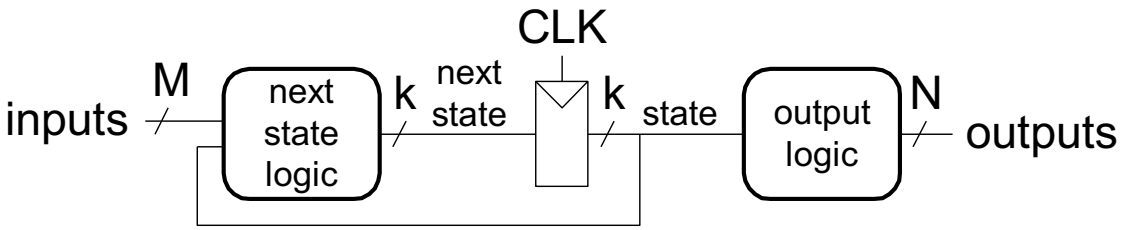
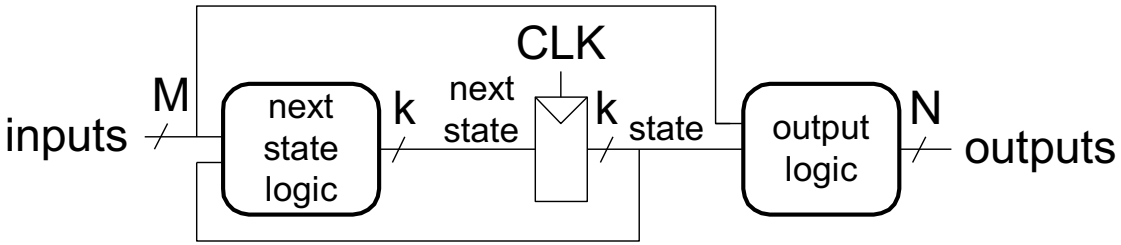
  - **Moore FSM**: outputs depend only on the current state
  - **Mealy FSM**: outputs depend on the current state and the inputs

Moore FSM

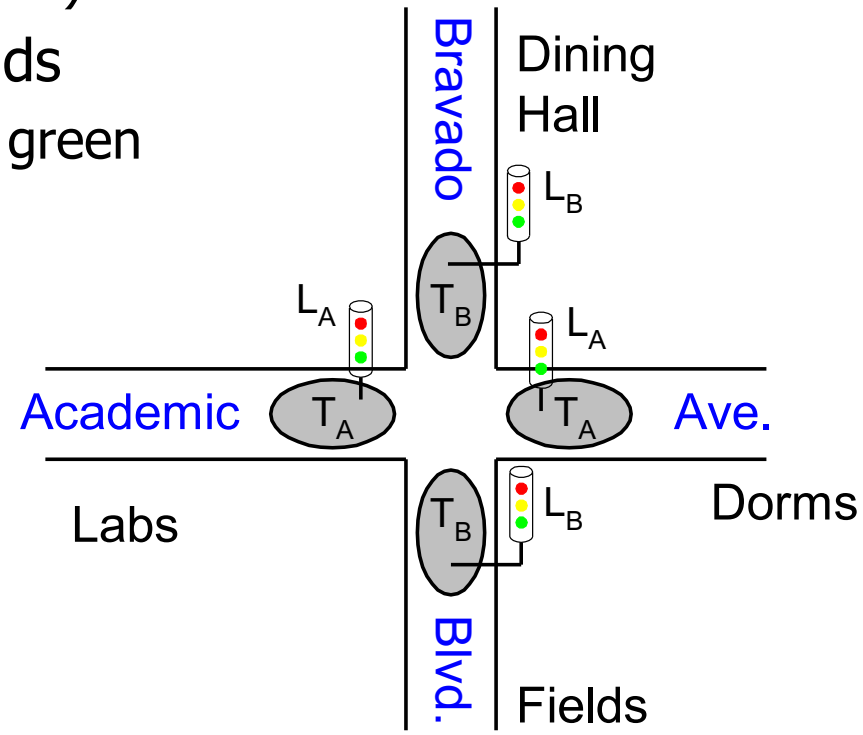


Mealy FSM



Finite State Machine Example

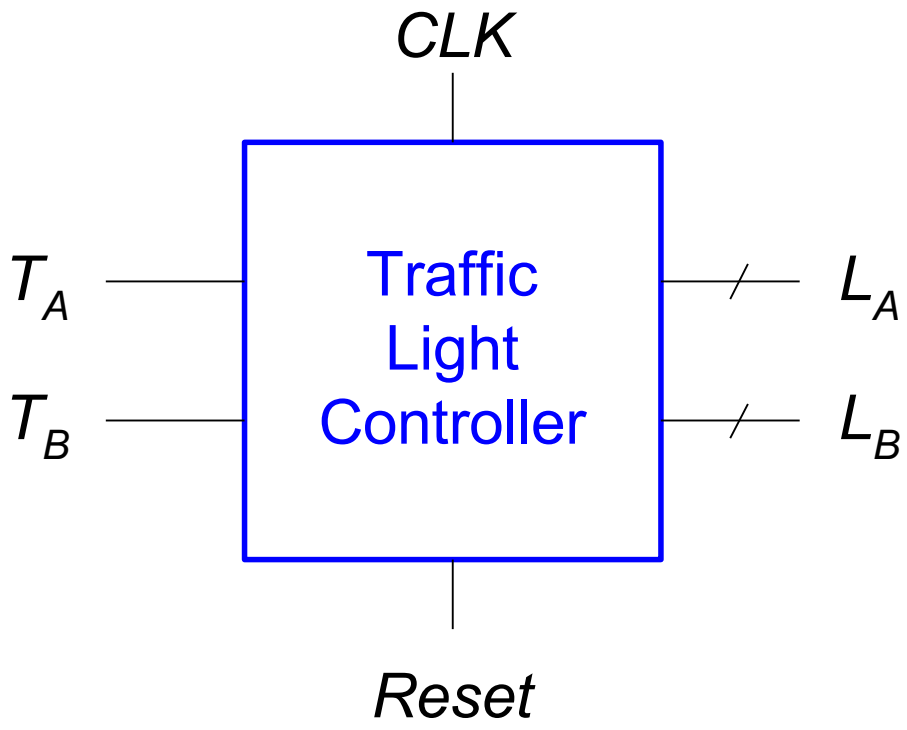
- "Smart" traffic light controller
  - **2 inputs**:
    - Traffic sensors: T_A , T_B (TRUE when there's traffic)
  - **2 outputs**:
    - Lights: L_A , L_B (Red, Yellow, Green)
  - State can change every 5 seconds
    - Except if green and traffic, stay green



From H&H Section 3.4.1

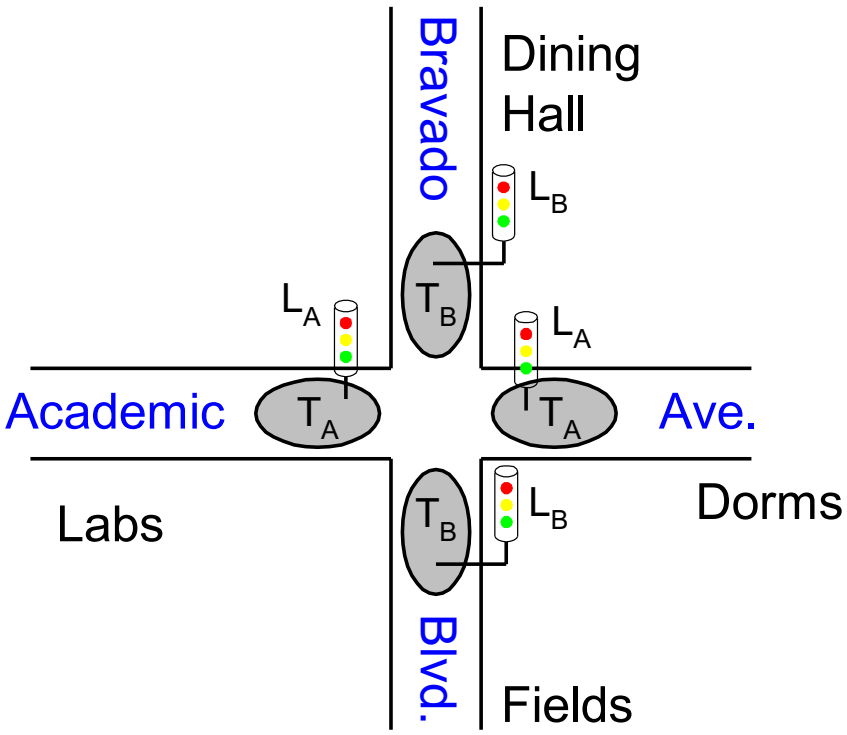
Finite State Machine Black Box

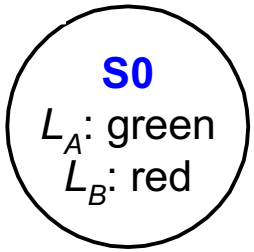
- **Inputs:** CLK, Reset, T_A , T_B
- **Outputs:** L_A , L_B



Finite State Machine Transition Diagram

- **Moore FSM:** outputs labeled in each state
  - **States:** Circles
  - **Transitions:** Arcs





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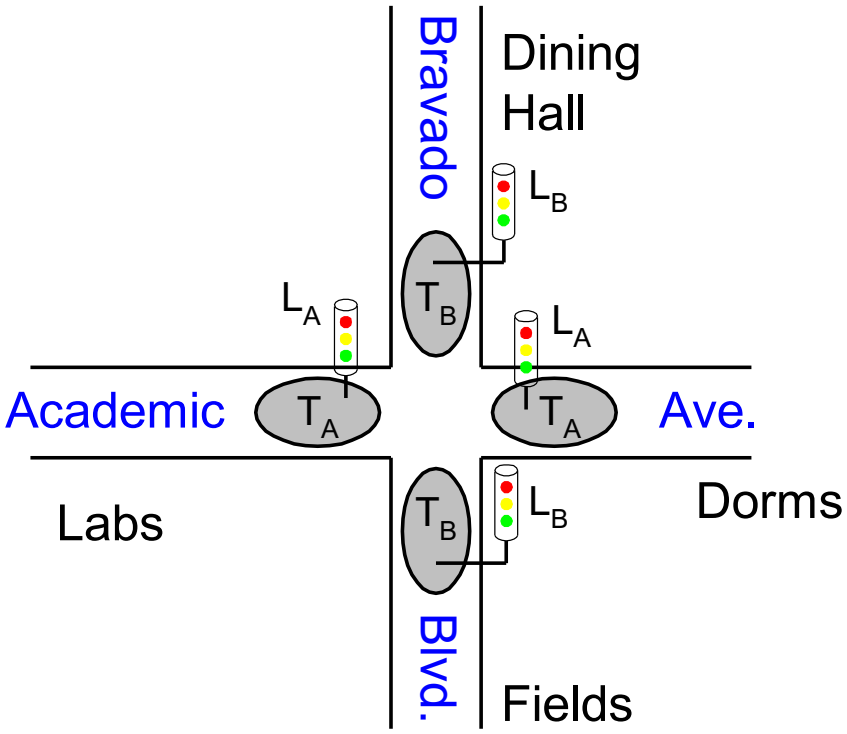

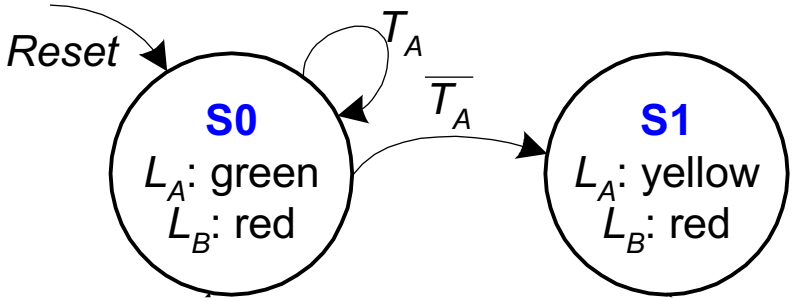

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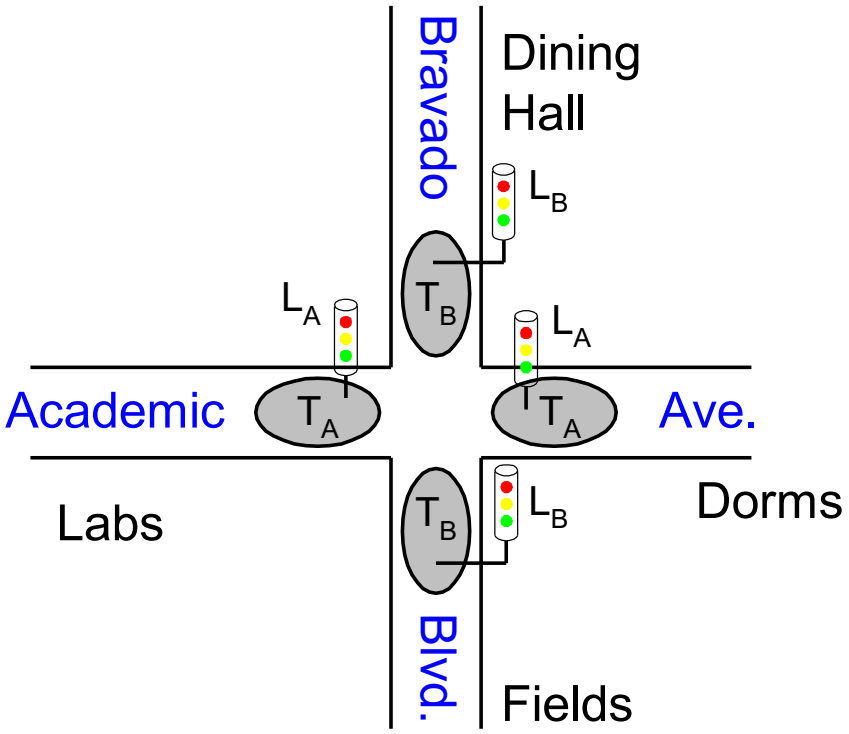

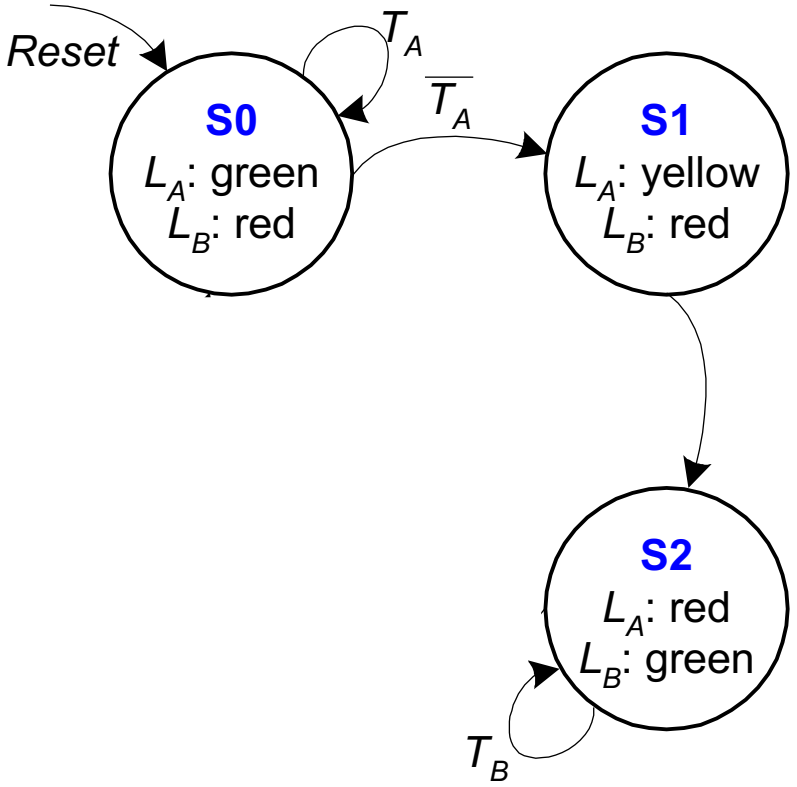

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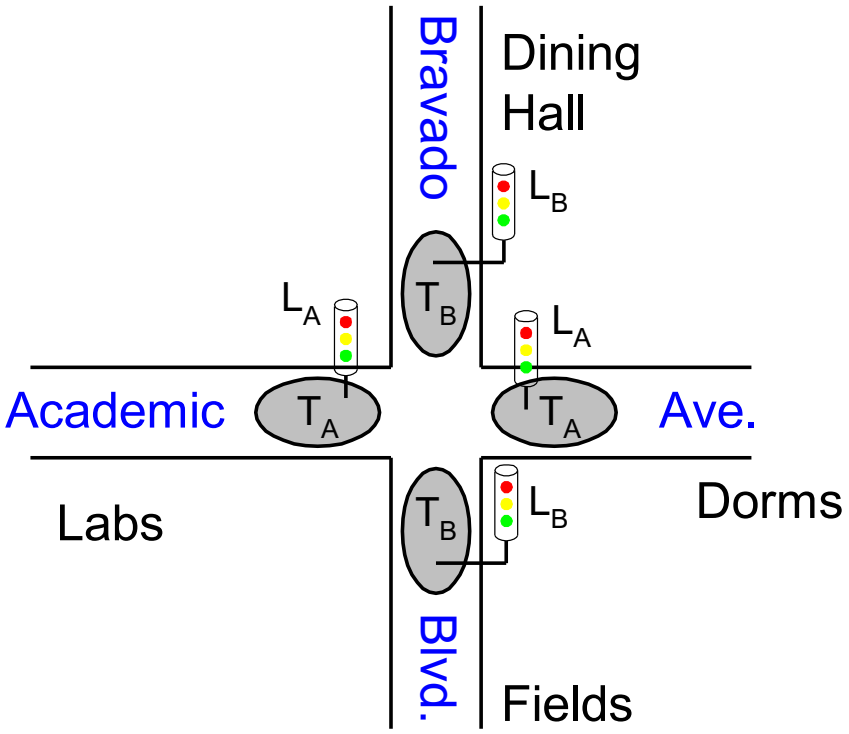

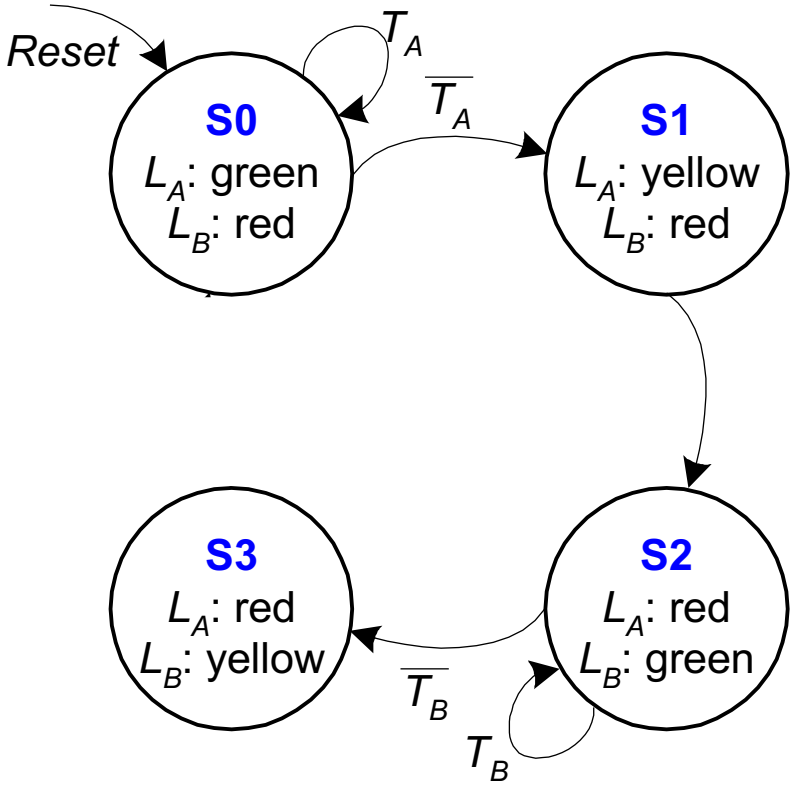

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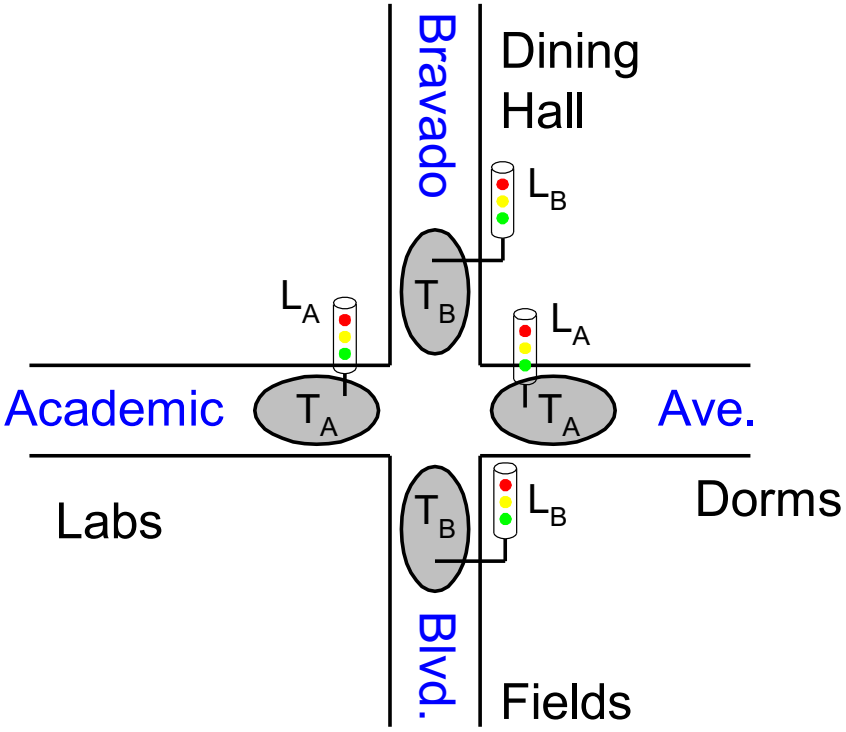
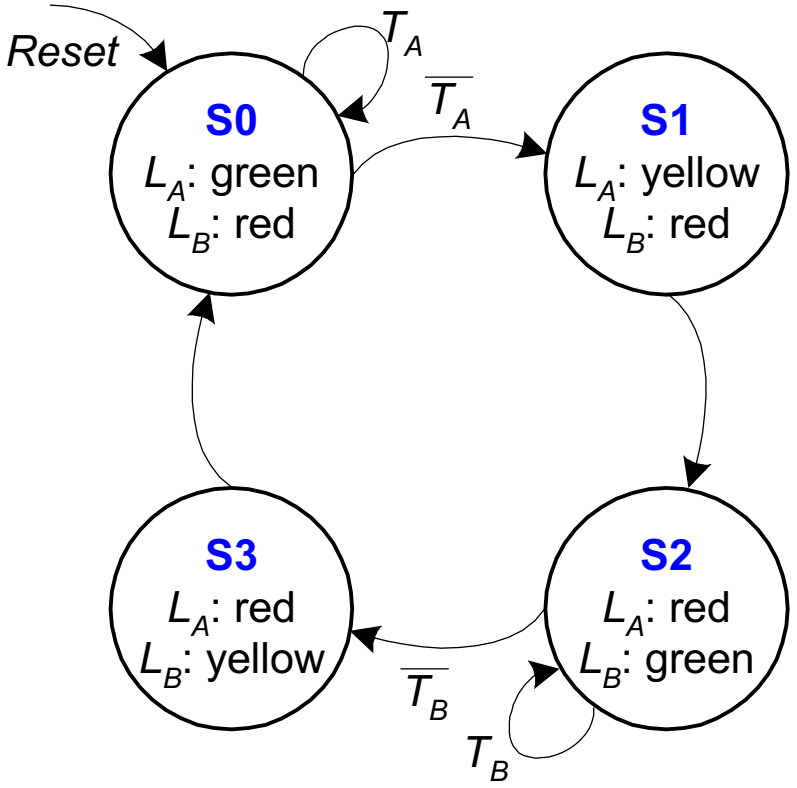
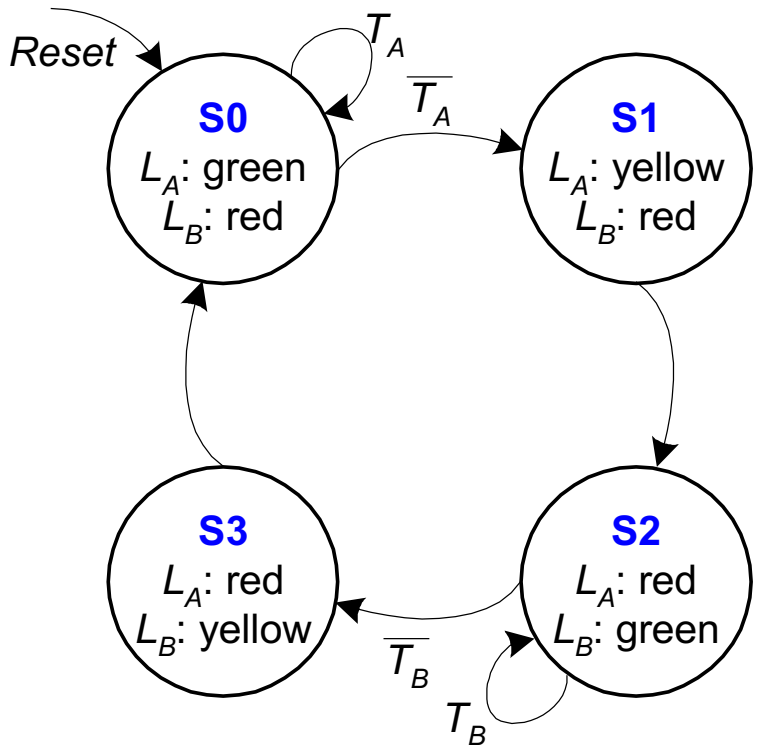
  - **States:** Circles
  - **Transitions:** Arcs





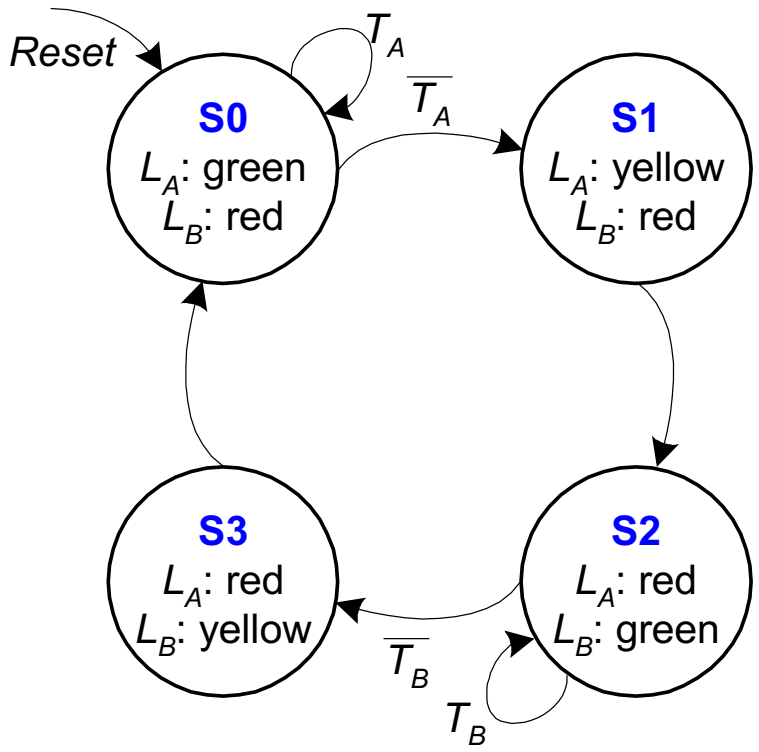
Finite State Machine: State Transition Table

FSM State Transition Table



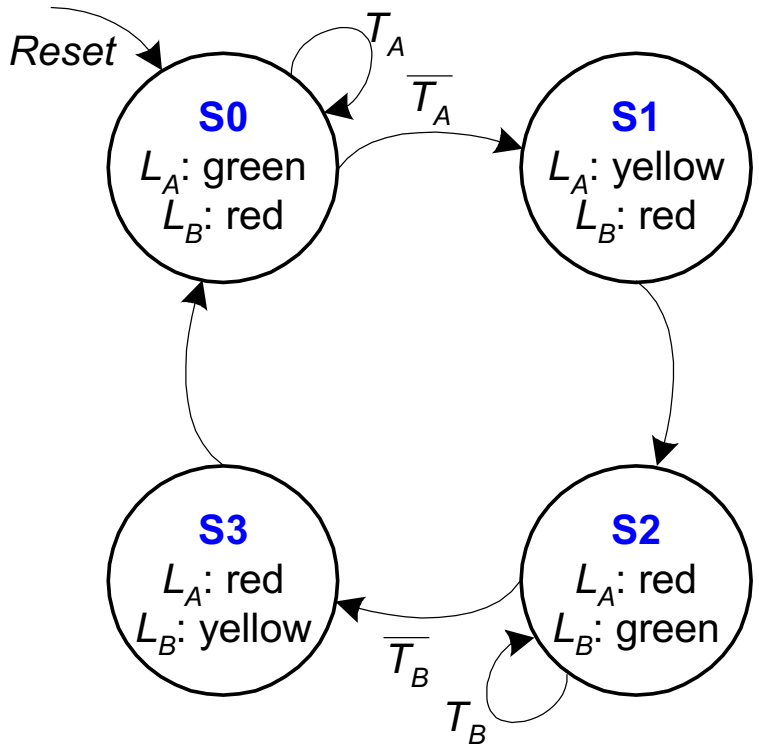
Current State	Inputs		Next State
S	T_A	T_B	S'
S0	0	X	
S0	1	X	
S1	X	X	
S2	X	0	
S2	X	1	
S3	X	X	

FSM State Transition Table



Current State	Inputs		Next State
S	T_A	T_B	S'
S0	0	X	S1
S0	1	X	S0
S1	X	X	S2
S2	X	0	S3
S2	X	1	S2
S3	X	X	S0

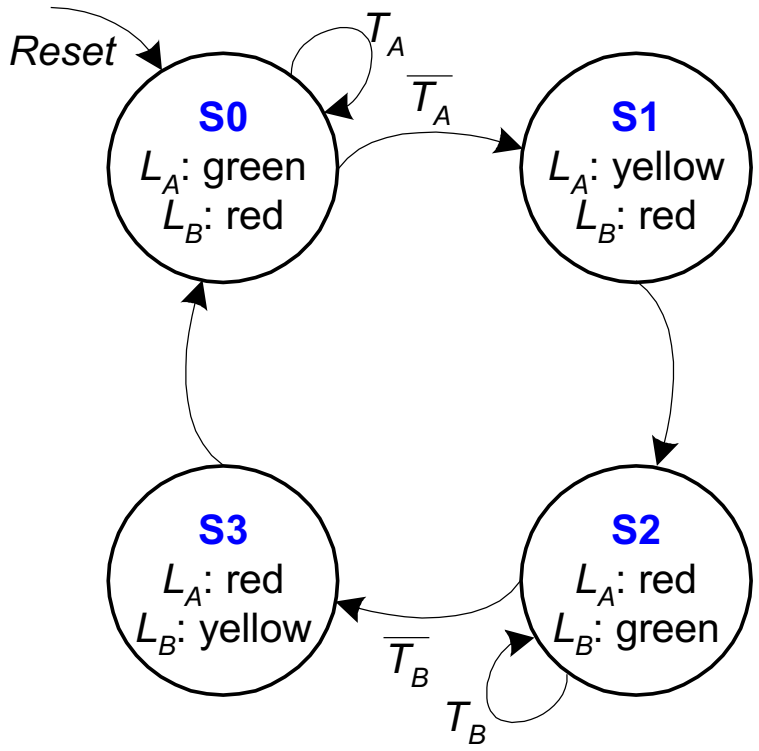
FSM State Transition Table



Current State	Inputs		Next State
S	T_A	T_B	S'
S0	0	X	S1
S0	1	X	S0
S1	X	X	S2
S2	X	0	S3
S2	X	1	S2
S3	X	X	S0

State	Encoding
S0	00
S1	01
S2	10
S3	11

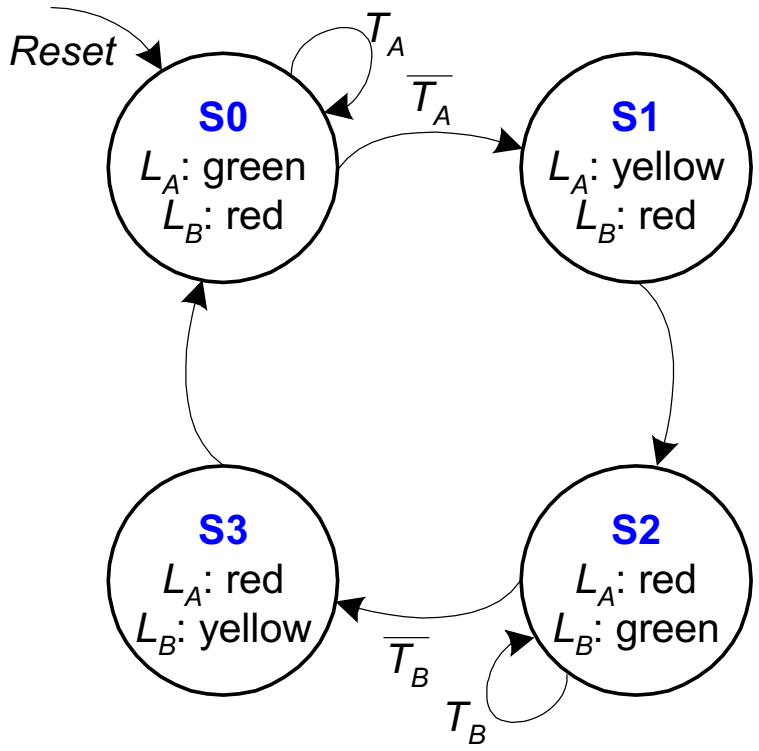
FSM State Transition Table



Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

FSM State Transition Table

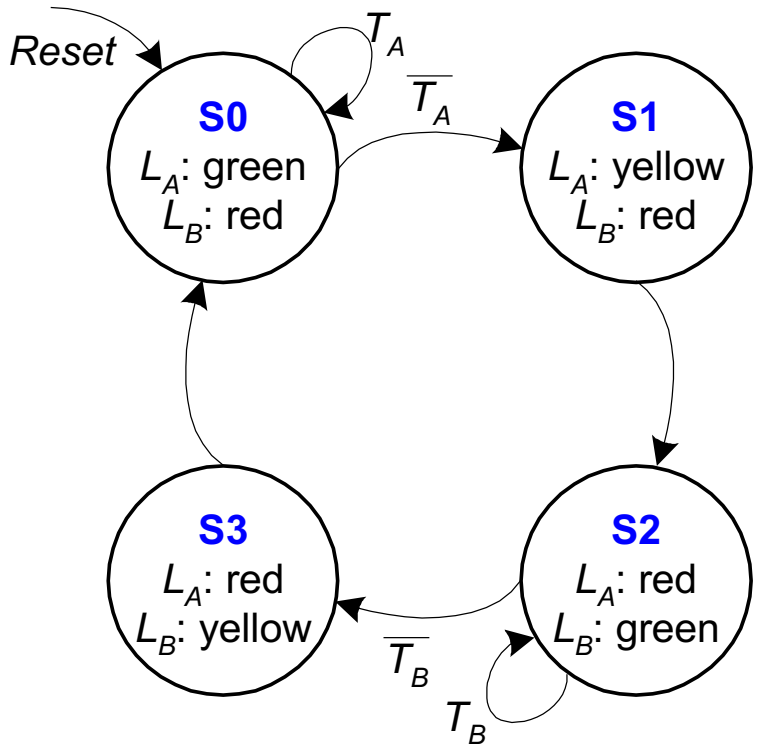


Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$S'_1 = ?$

FSM State Transition Table

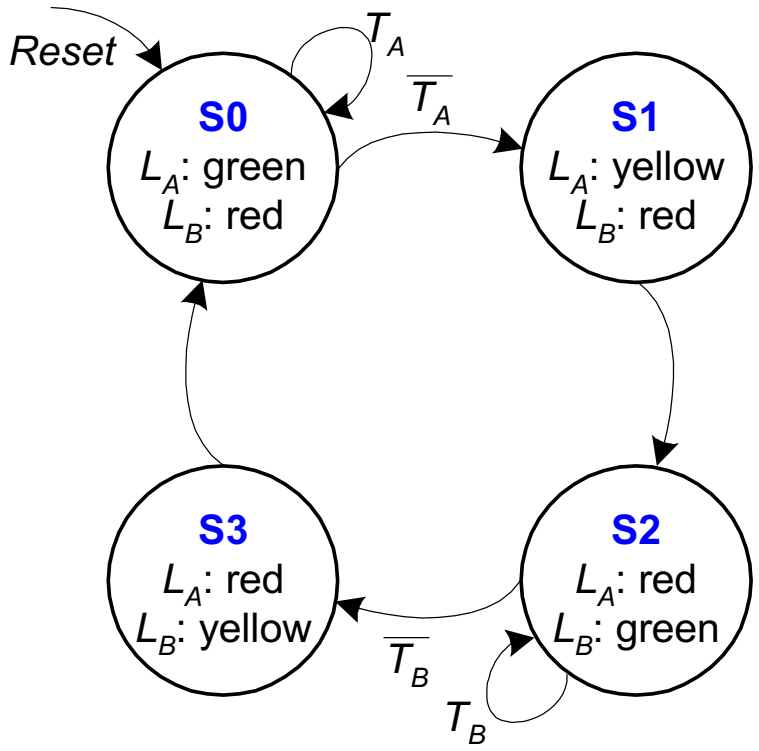


Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$$S'_1 = (\overline{S_1} \cdot S_0) + (S_1 \cdot \overline{S_0} \cdot \overline{T_B}) + (S_1 \cdot \overline{S_0} \cdot T_B)$$

FSM State Transition Table



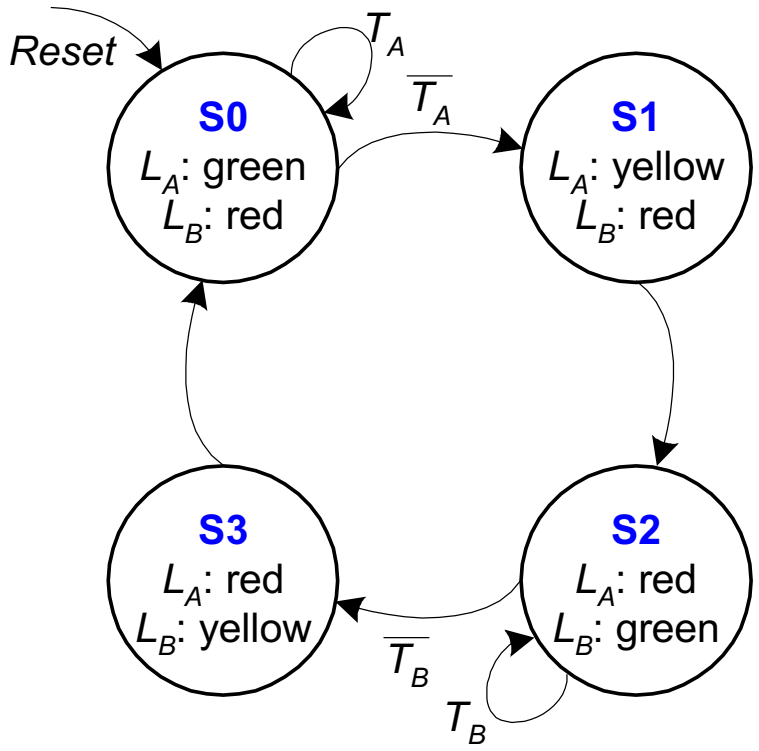
Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$$S'_1 = (\overline{S_1} \cdot S_0) + (S_1 \cdot \overline{S_0} \cdot \overline{T_B}) + (S_1 \cdot \overline{S_0} \cdot T_B)$$

$$S'_0 = ?$$

FSM State Transition Table



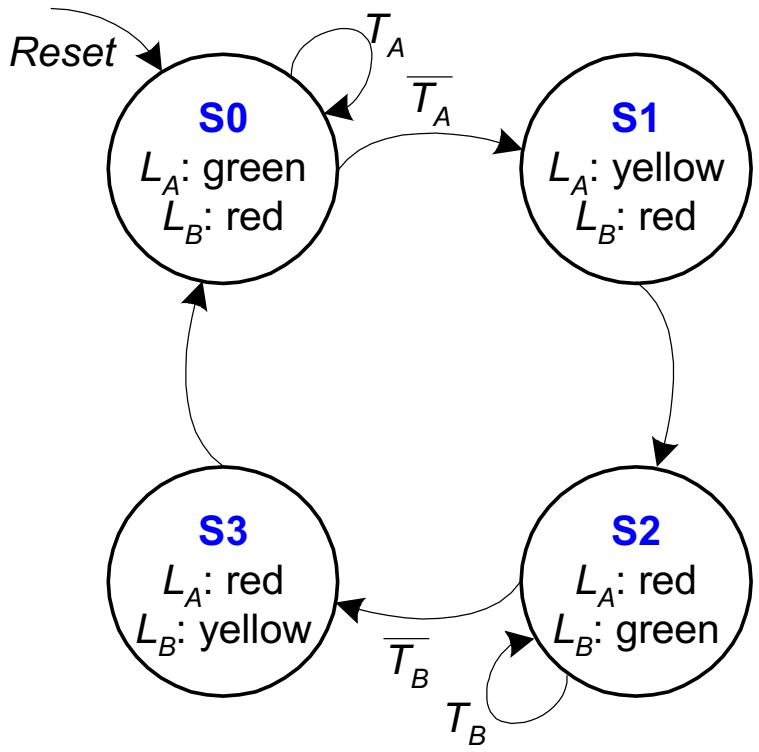
Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$$S'_1 = (\overline{S_1} \cdot S_0) + (S_1 \cdot \overline{S_0} \cdot \overline{T_B}) + (S_1 \cdot \overline{S_0} \cdot T_B)$$

$$S'_0 = (\overline{S_1} \cdot \overline{S_0} \cdot \overline{T_A}) + (S_1 \cdot \overline{S_0} \cdot \overline{T_B})$$

FSM State Transition Table



Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

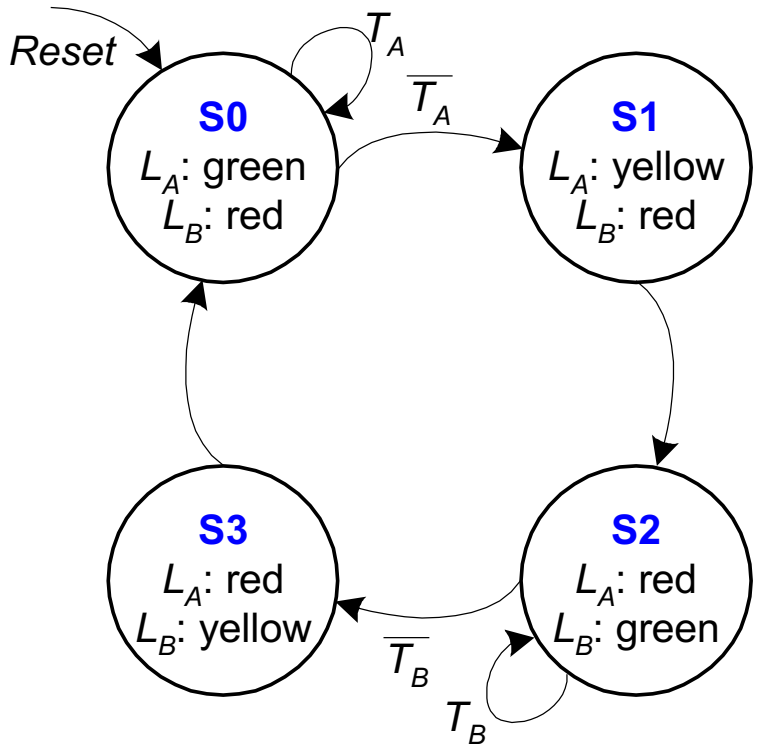
State	Encoding
S0	00
S1	01
S2	10
S3	11

$S'_1 = S_1 \text{ xor } S_0$ **(Simplified)**

$S'_0 = (\overline{S_1} \cdot \overline{S_0} \cdot \overline{T_A}) + (S_1 \cdot \overline{S_0} \cdot \overline{T_B})$

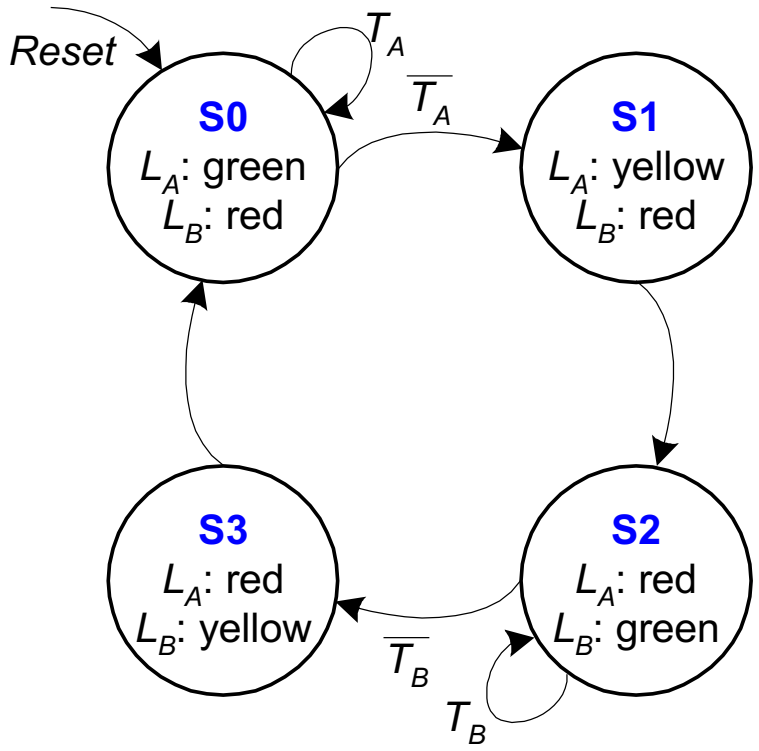
Finite State Machine: Output Table

FSM Output Table



Current State		Outputs	
S_1	S_0	L_A	L_B
0	0	green	red
0	1	yellow	red
1	0	red	green
1	1	red	yellow

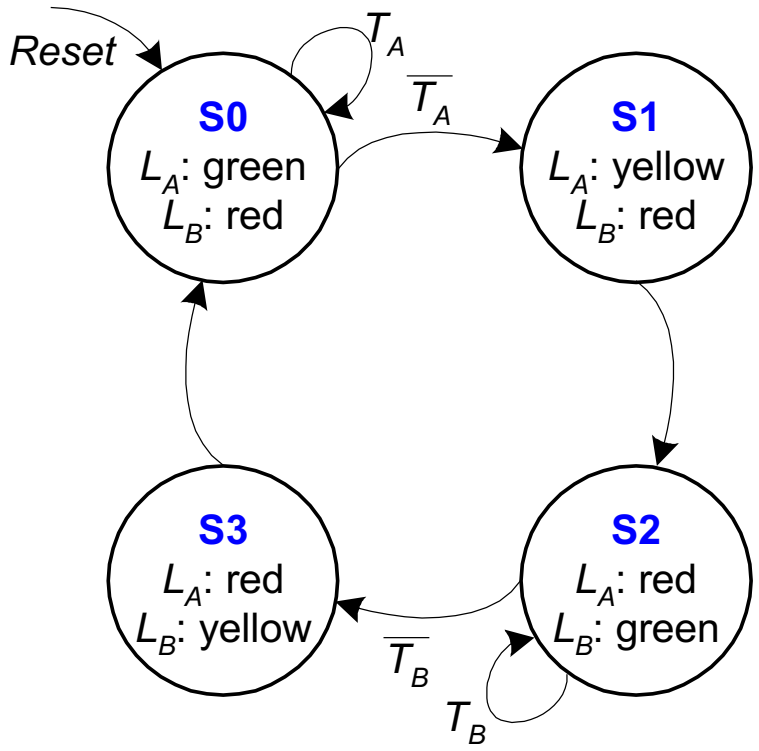
FSM Output Table



Current State		Outputs	
S_1	S_0	L_A	L_B
0	0	green	red
0	1	yellow	red
1	0	red	green
1	1	red	yellow

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table

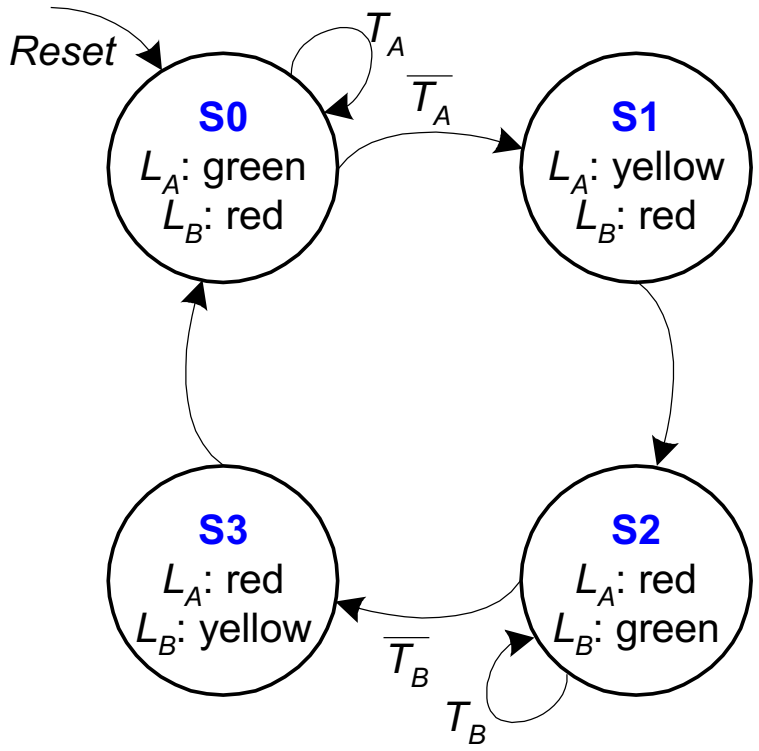


Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

$L_{A1} = S_1$

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table

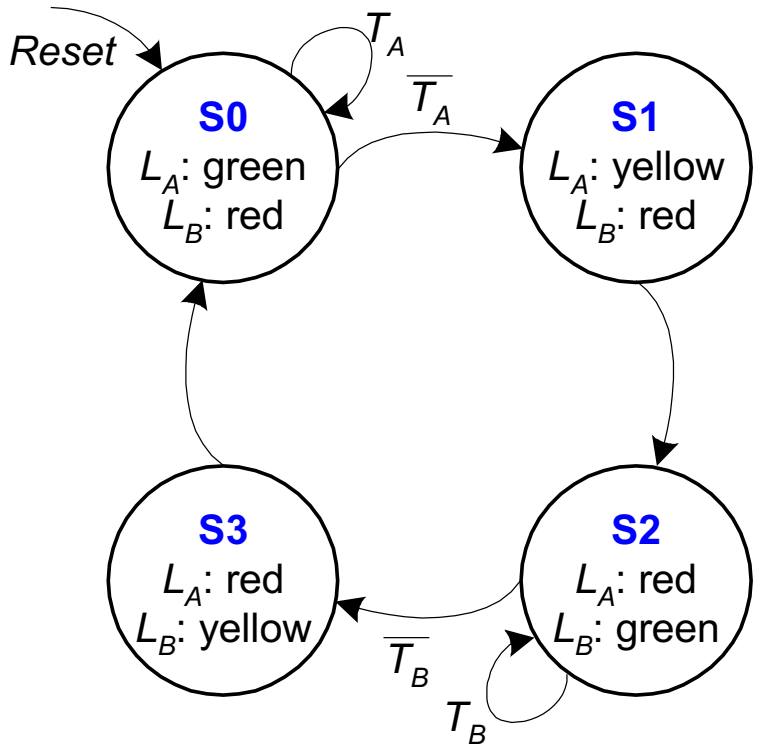


Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

$$L_{A1} = S_1$$
$$L_{A0} = \overline{S_1} \cdot S_0$$

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table

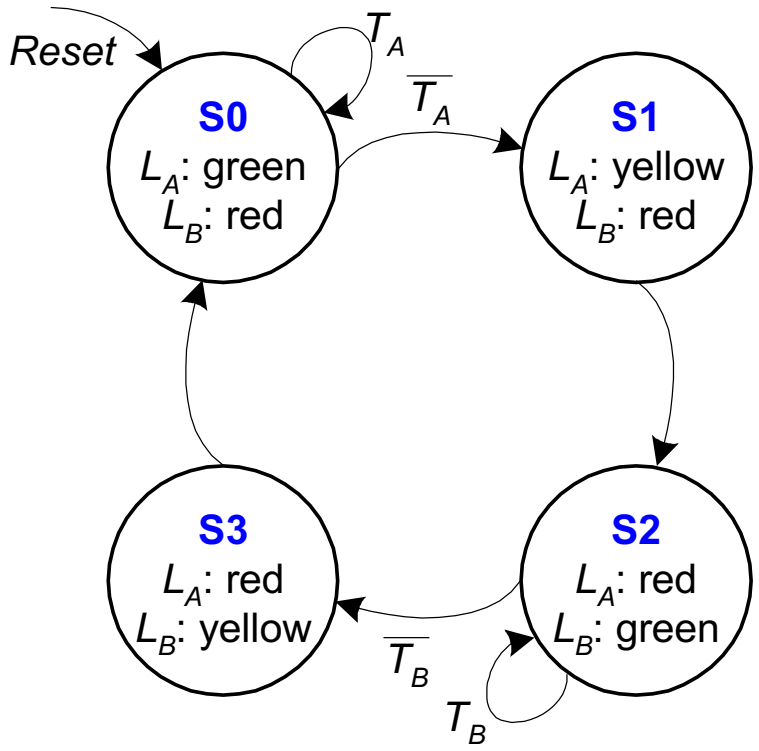


Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

$$L_{A1} = S_1$$
$$L_{A0} = \overline{S_1} \cdot S_0$$
$$L_{B1} = \overline{S_1}$$

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table



Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

Output	Encoding
green	00
yellow	01
red	10

$$L_{A1} = S_1$$
$$L_{A0} = \overline{S_1} \cdot S_0$$
$$L_{B1} = \overline{S_1}$$
$$L_{B0} = S_1 \cdot S_0$$

Digital Design & Computer Arch.

Lecture 6: Sequential Logic Design

Prof. Onur Mutlu

ETH Zürich

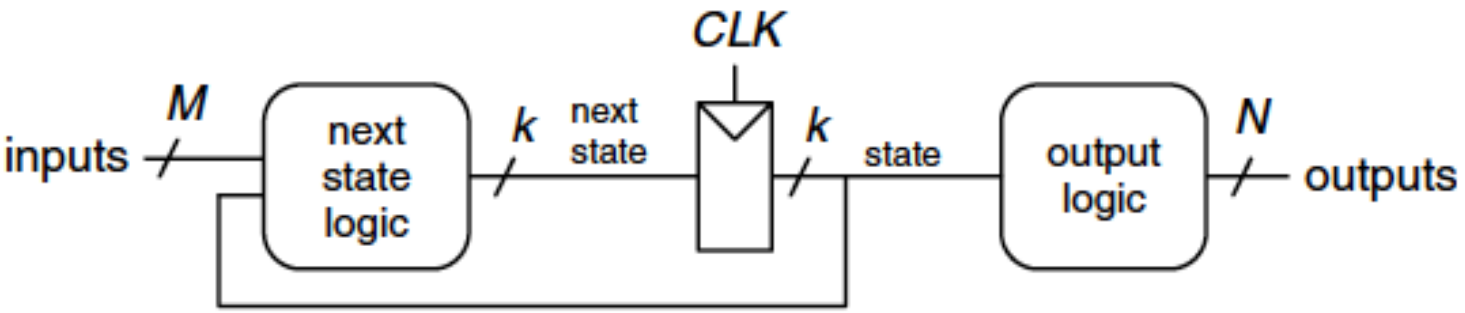
Spring 2020

6 March 2020

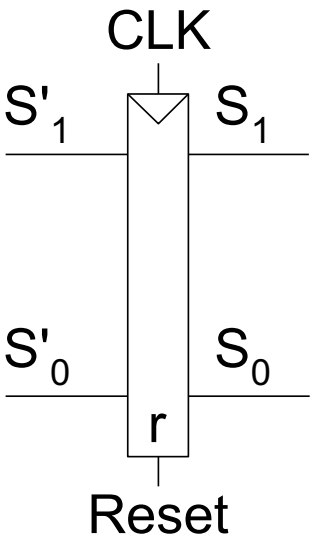
We did not cover the remaining slides. They are for your preparation for the next lecture.

Finite State Machine: Schematic

FSM Schematic: State Register

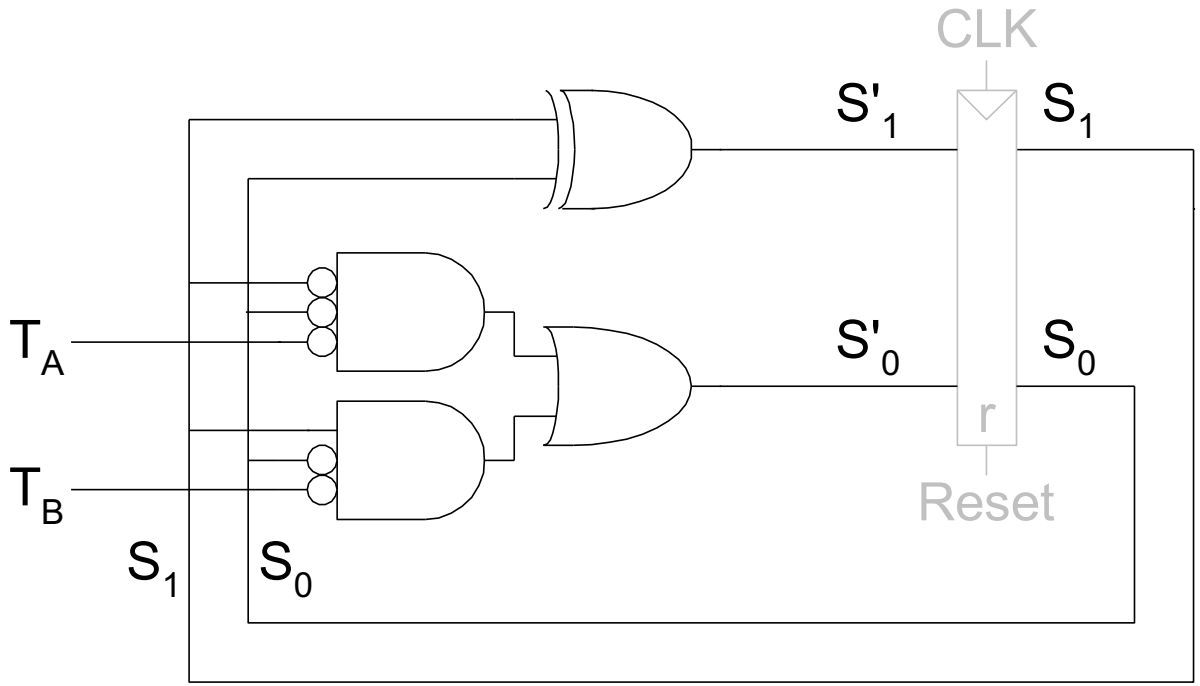


FSM Schematic: State Register



state register

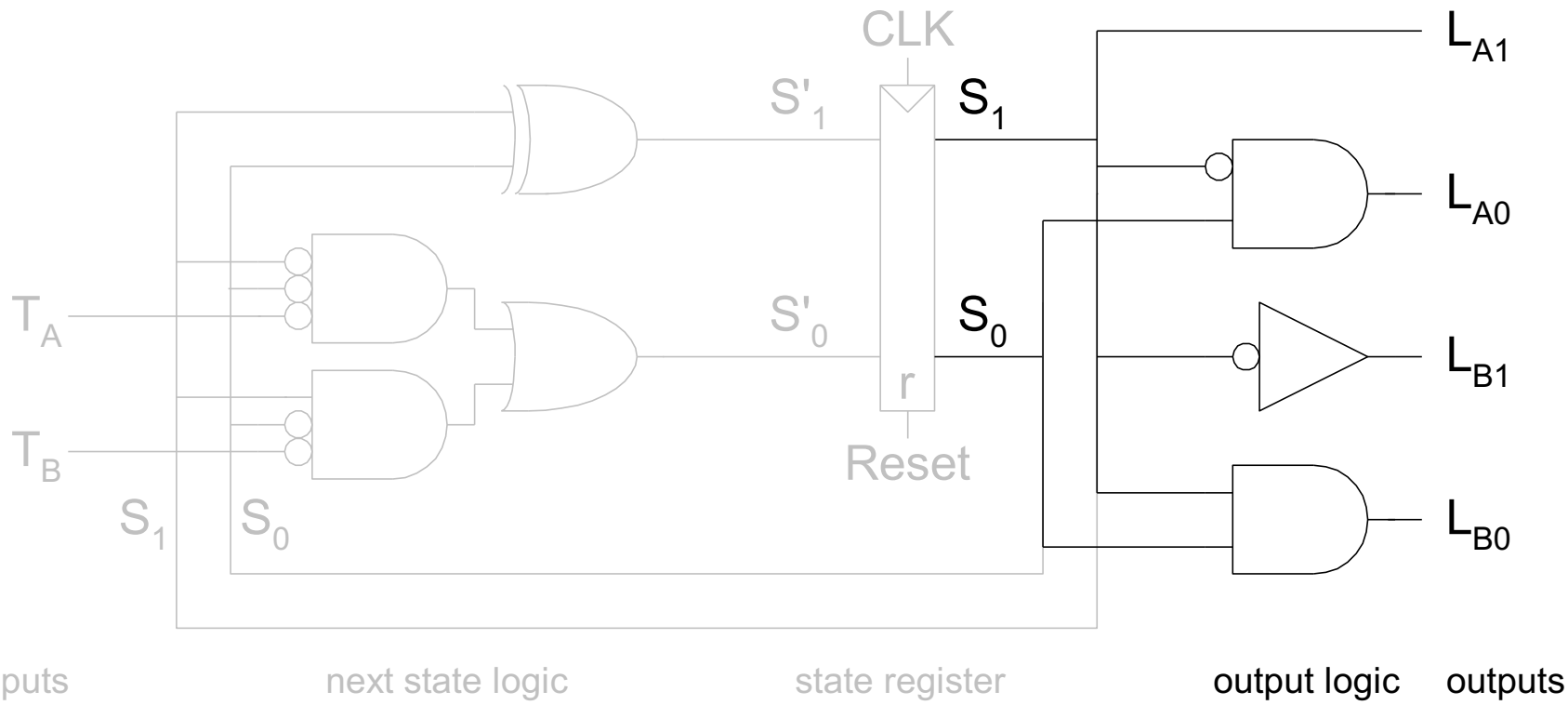
FSM Schematic: Next State Logic



$$S'_1 = S_1 \text{ xor } S_0$$

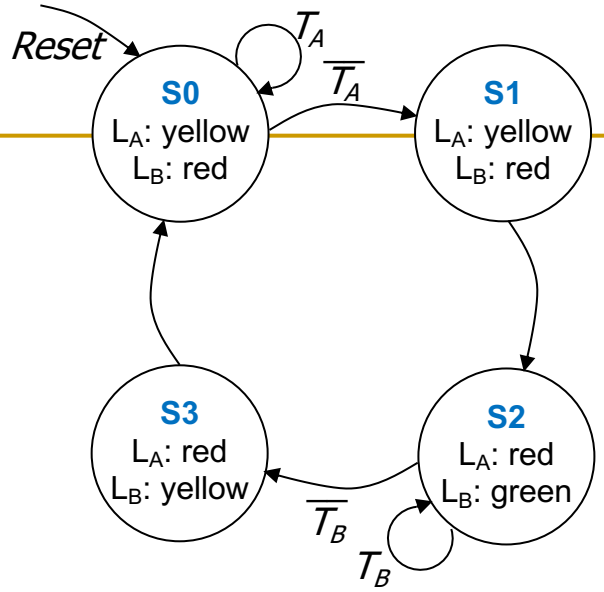
$$S'_0 = (\overline{S}_1 \cdot \overline{S}_0 \cdot \overline{T}_A) + (S_1 \cdot \overline{S}_0 \cdot \overline{T}_B)$$

FSM Schematic: Output Logic

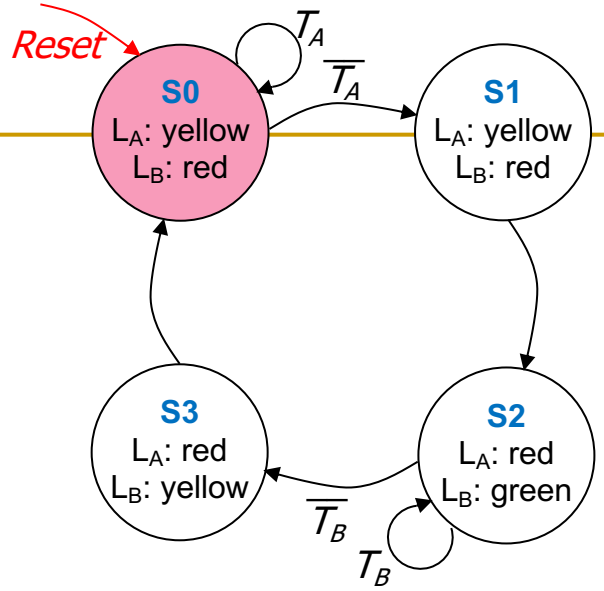


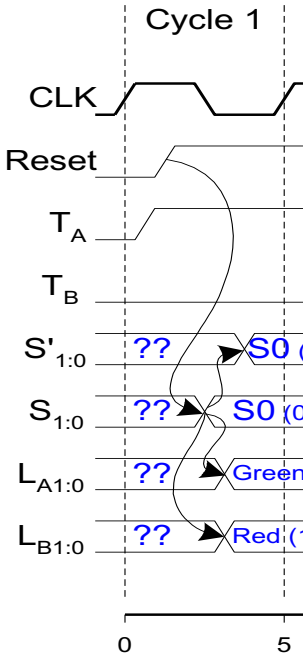
$$L_{A1} = S_1$$
$$L_{A0} = \overline{S_1} \cdot S_0$$
$$L_{B1} = \overline{S_1}$$
$$L_{B0} = S_1 \cdot S_0$$

FSM Timing Diagram

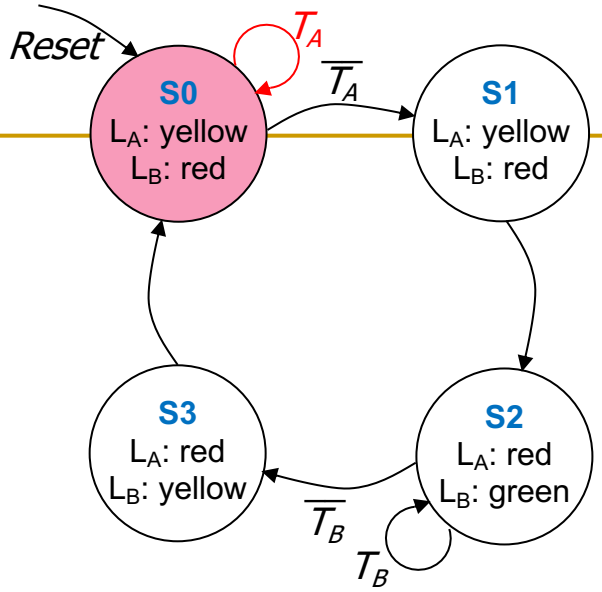


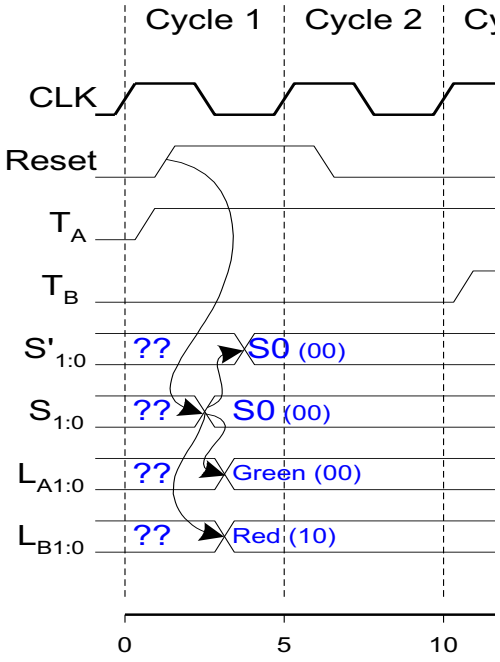
FSM Timing Diagram



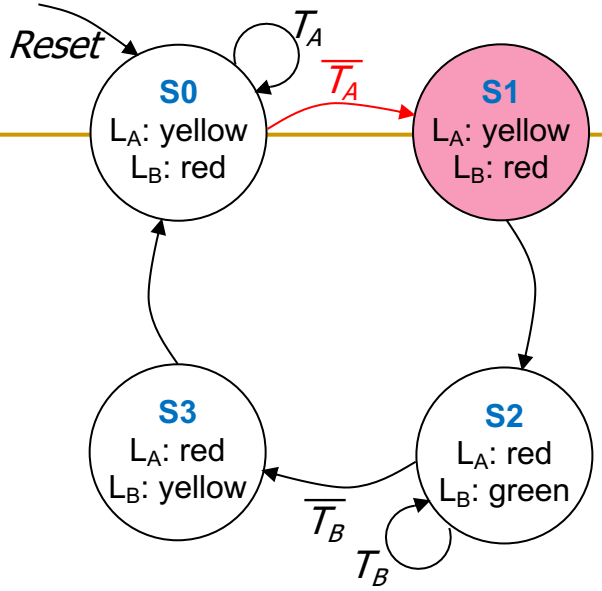


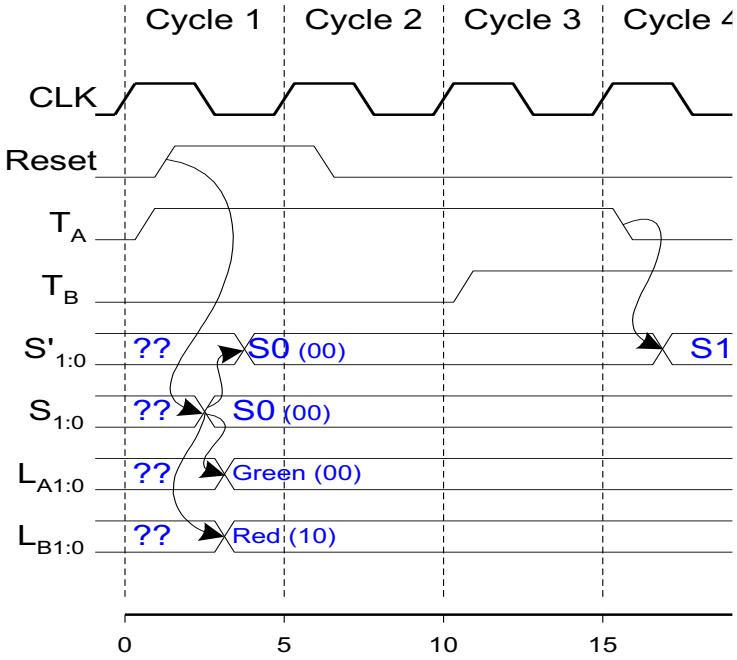
FSM Timing Diagram



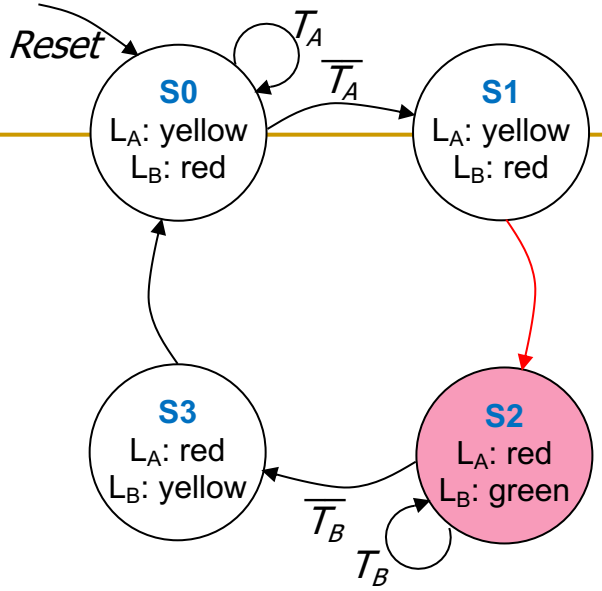


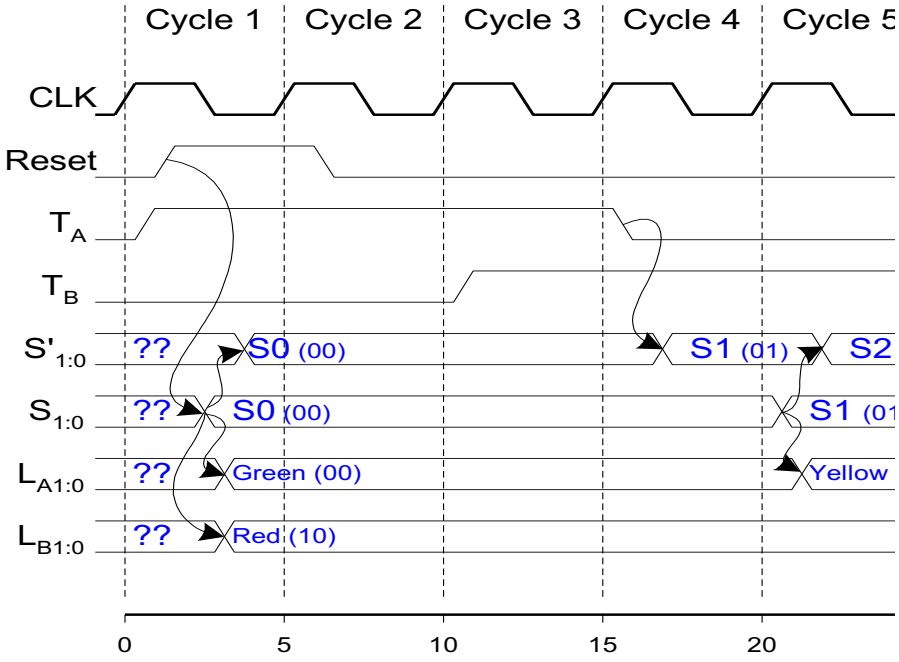
FSM Timing Diagram



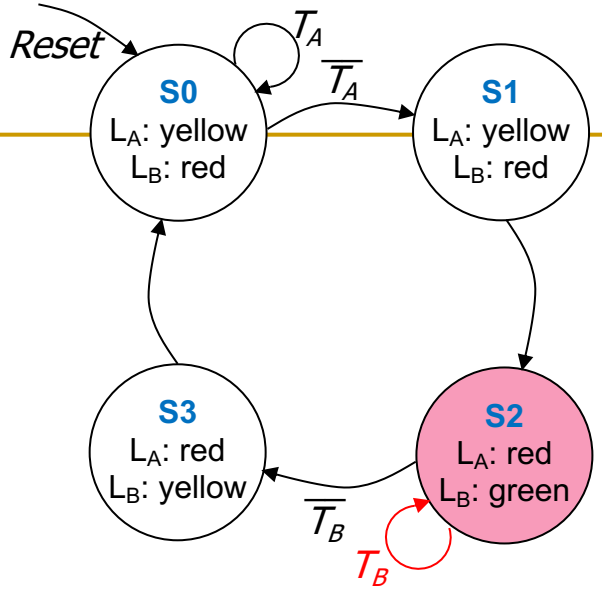


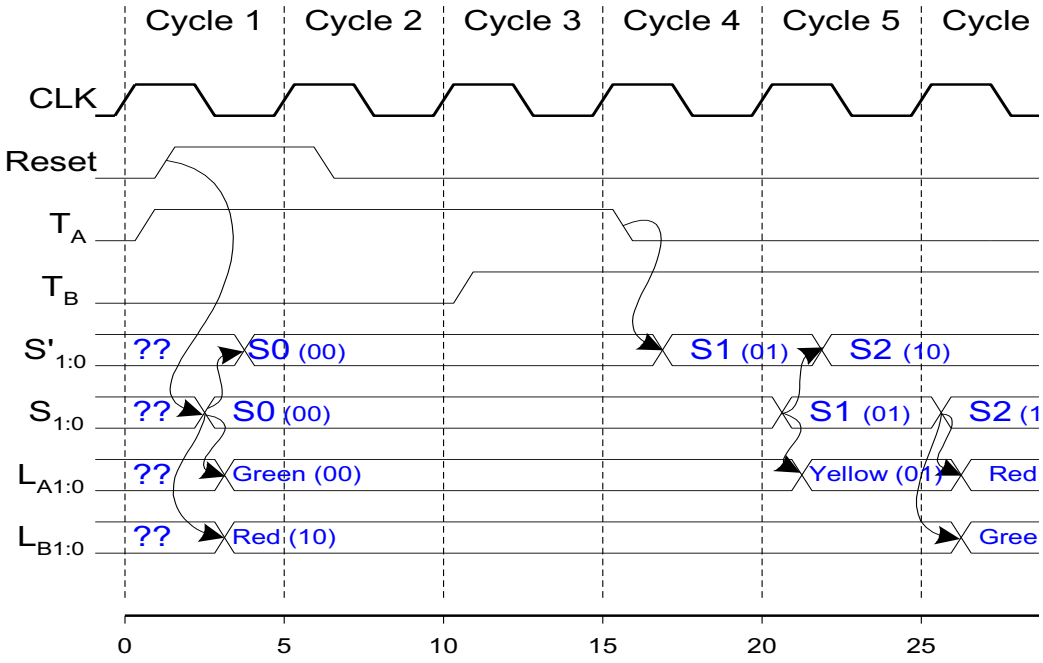
FSM Timing Diagram



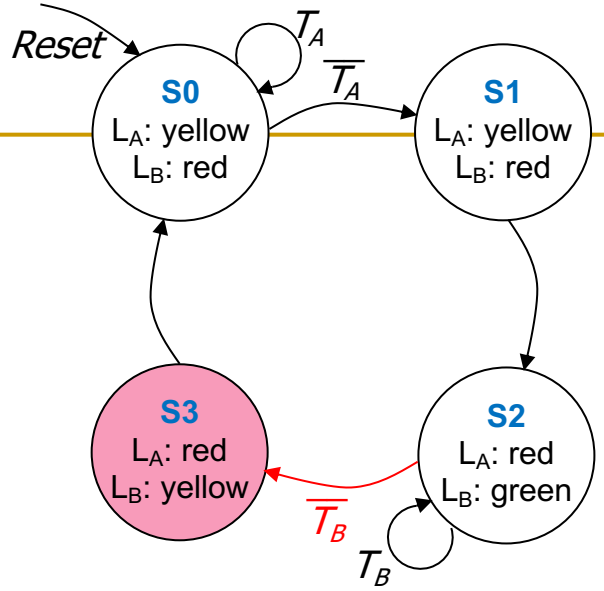


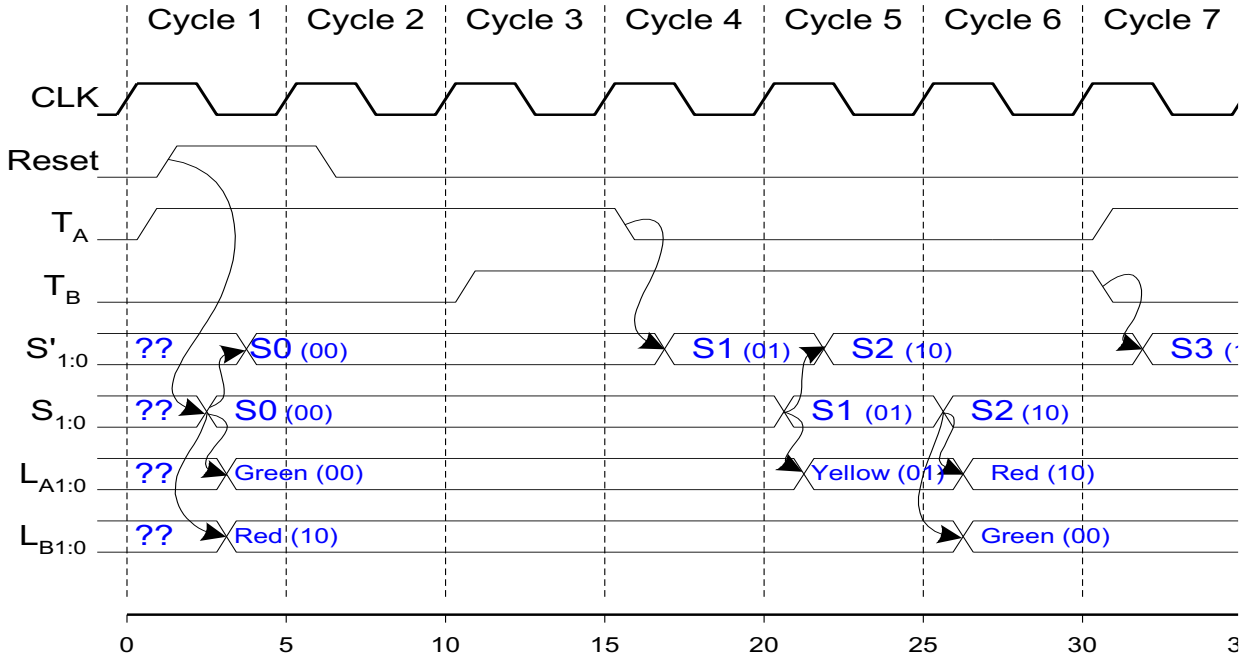
FSM Timing Diagram



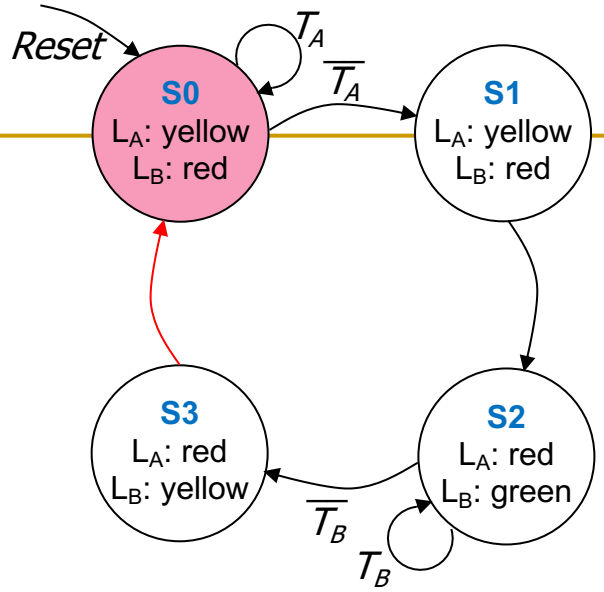


FSM Timing Diagram

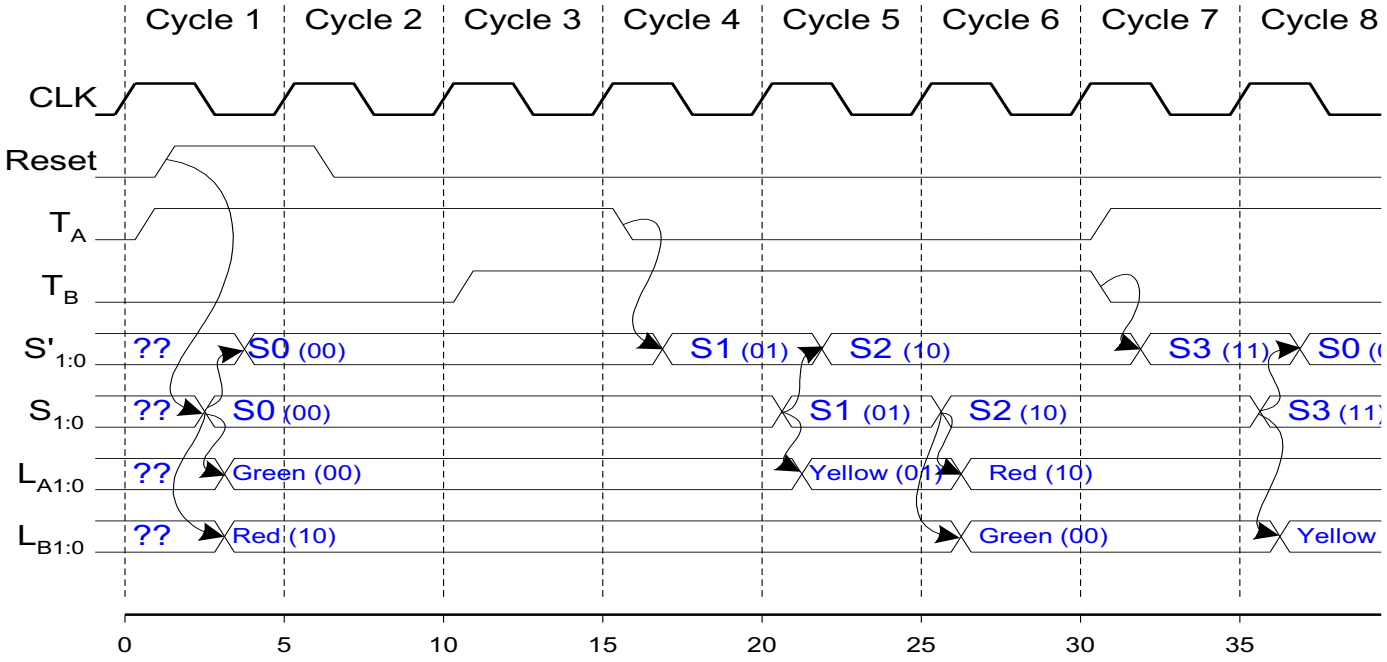




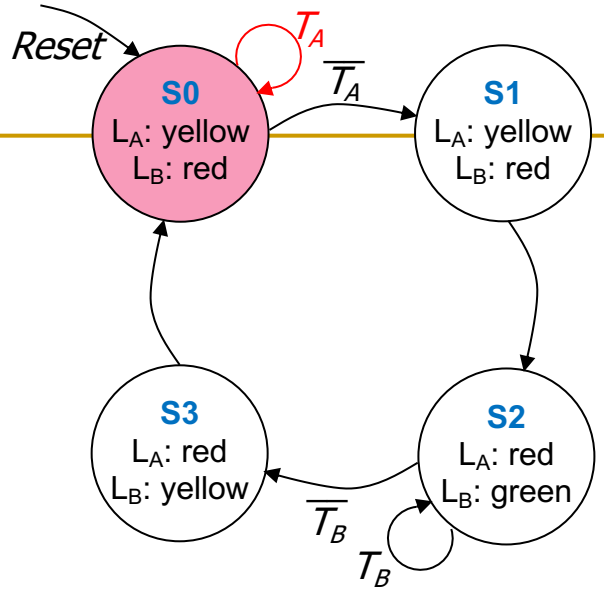
FSM Timing Diagram

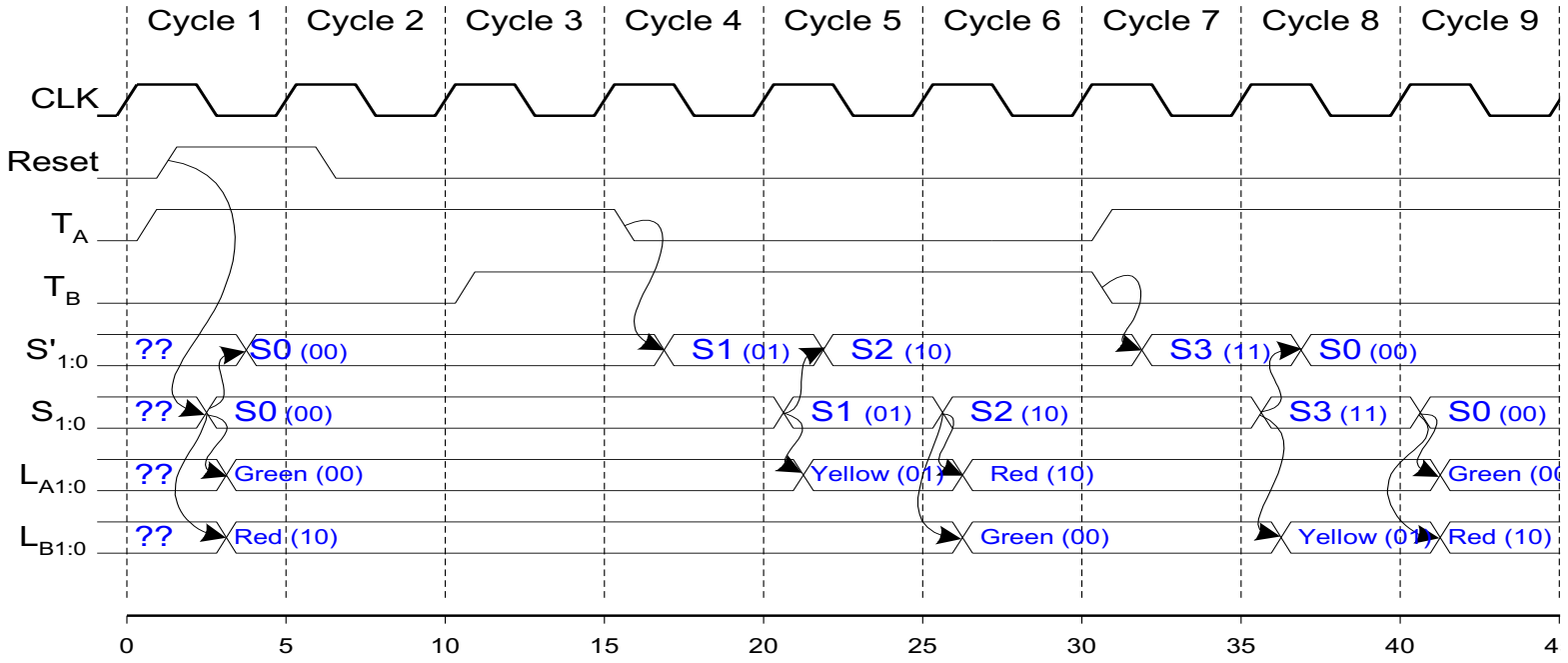


This is from H&H Section 3.4.1

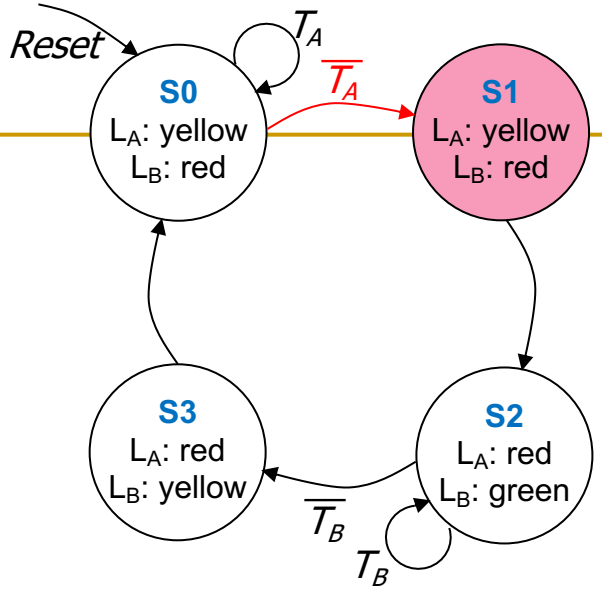


FSM Timing Diagram

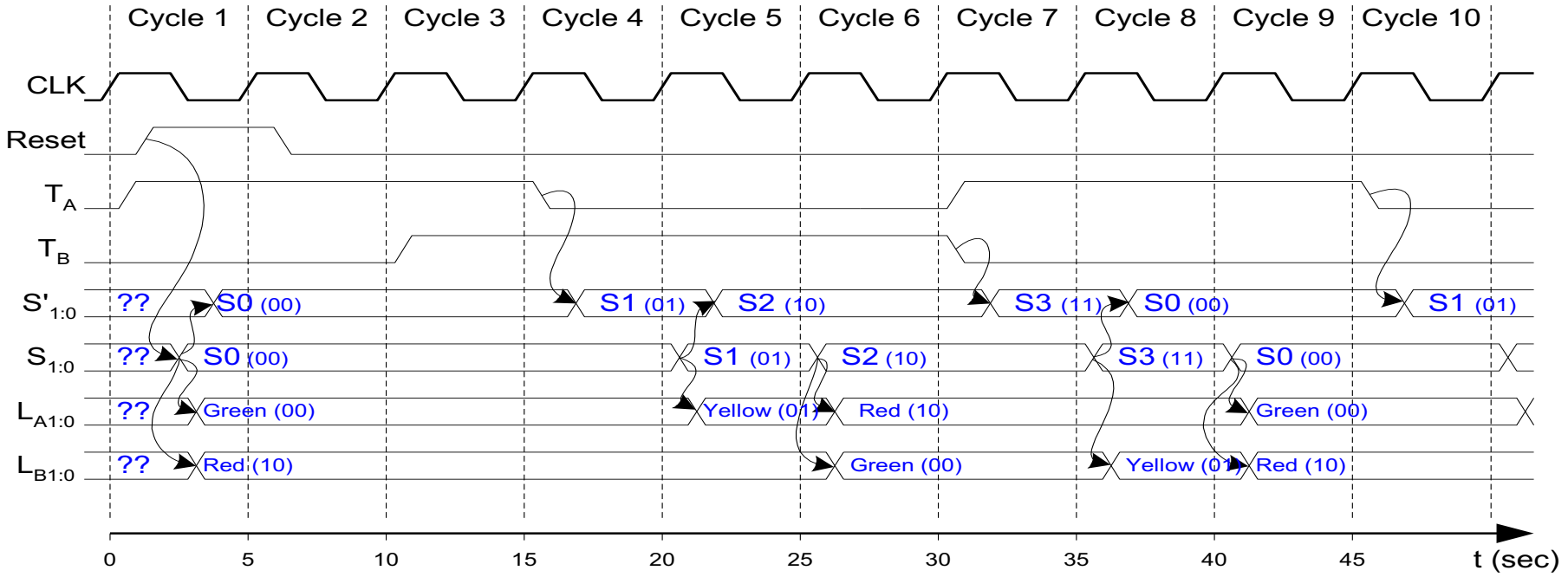




FSM Timing Diagram



See H&H Chapter 3.4

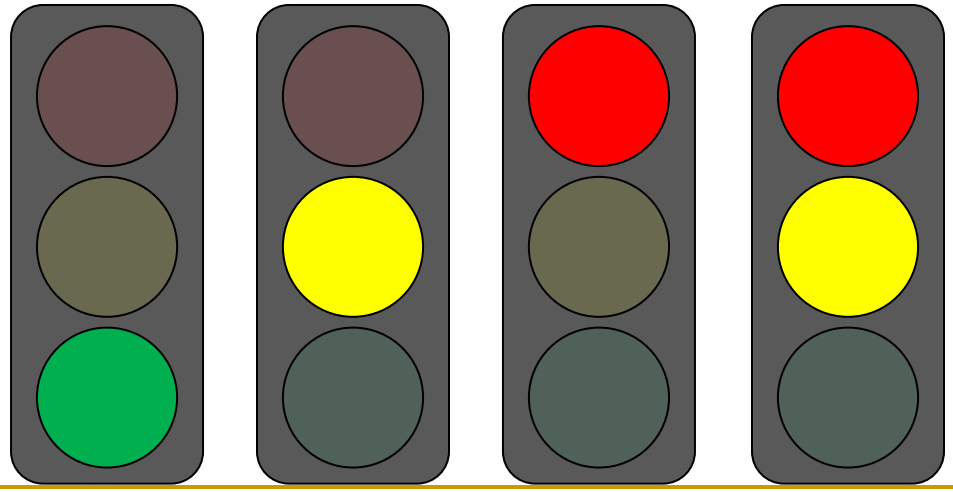


Finite State Machine: State Encoding

FSM State Encoding

- How do we encode the state bits?
  - Three common state binary encodings with different tradeoffs
    1. **Fully Encoded**
    2. **1-Hot Encoded**
    3. **Output Encoded**

- Let's see an example **Swiss** traffic light with 4 states
  - Green, Yellow, Red, Yellow+Red



FSM State Encoding (II)

1. **Binary Encoding (Full Encoding):**
   - Use the minimum number of bits used to encode all states
     - Use $log_2(num_states)$ bits to represent the states
   - *Example states:* 00, 01, 10, 11
   - **Minimizes** # flip-flops, but not necessarily output logic or next state logic

2. **One-Hot Encoding:**
   - Each bit encodes a different state
     - Uses *num_states* bits to represent the states
     - Exactly 1 bit is "hot" for a given state
   - *Example states:* 0001, 0010, 0100, 1000
   - **Simplest design process** – very automatable
   - **Maximizes** # flip-flops, **minimizes** next state logic

FSM State Encoding (III)

3. Output Encoding:

- Outputs are **directly accessible** in the state encoding
- For example, since we have **3 outputs** (light color), encode state with **3 bits**, where each bit represents a color
- *Example states:* 001, 010, 100, 110
  - Bit_0 encodes **green** light output,
  - Bit_1 encodes **yellow** light output
  - Bit_2 encodes **red** light output
- **Minimizes** output logic
- Only works for Moore Machines (output function of state)

FSM State Encoding (III)

3. **Output Encoding:**
   - Outputs are **directly accessible** in the state encoding

The **designer** must **carefully** choose an encoding scheme to **optimize** the design under given constraints

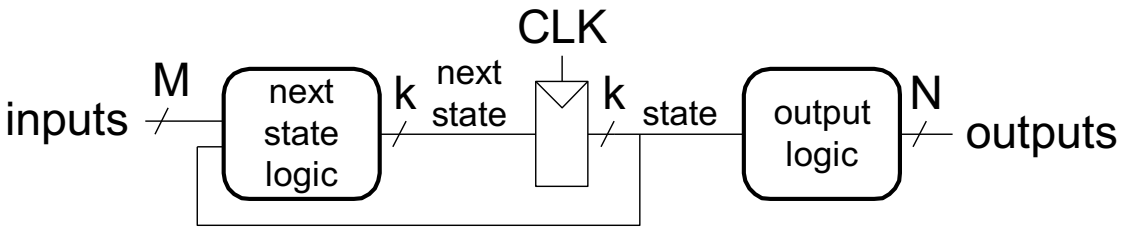
   - **Minimizes** output logic
   - Only works for Moore Machines (output function of state)

Moore vs. Mealy Machines

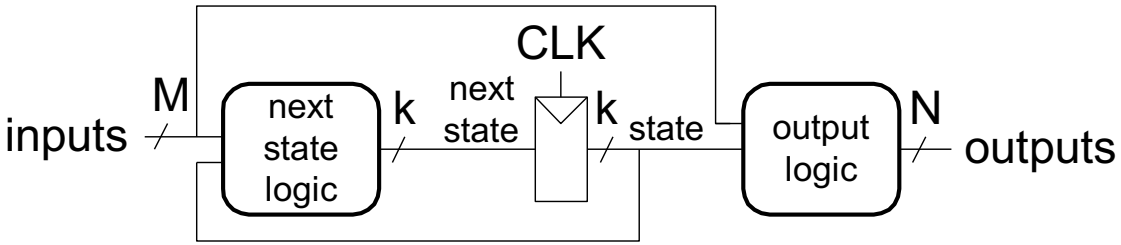
Recall: Moore vs. Mealy FSMs

- Next state is determined by the current state and the inputs
- Two types of finite state machines differ in the **output logic**:
  - **Moore FSM**: outputs depend only on the current state
  - **Mealy FSM**: outputs depend on the current state and the inputs

Moore FSM

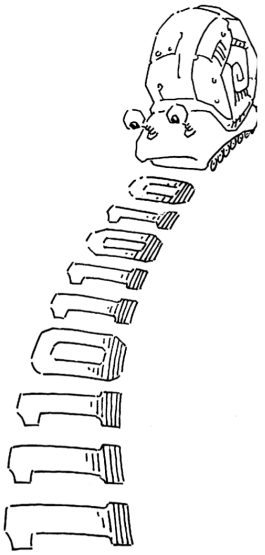


Mealy FSM

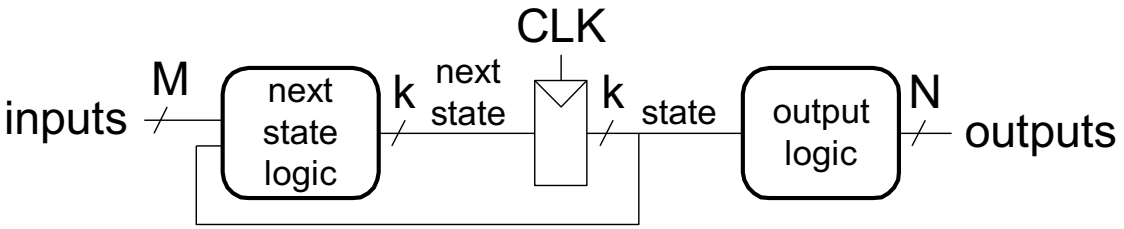


Moore vs. Mealy FSM Examples

- Alyssa P. Hacker has a snail that crawls down a paper tape with 1's and 0's on it.
- The snail smiles whenever the last four digits it has crawled over are 1101.
- Design Moore and Mealy FSMs of the snail's brain.



Moore FSM



Moore vs. Mealy FSM Examples

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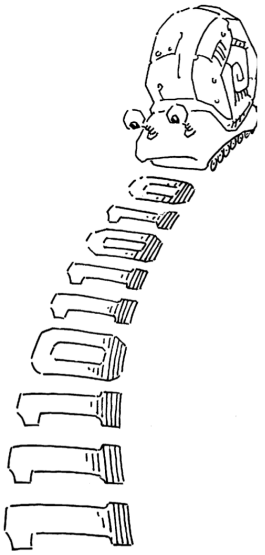
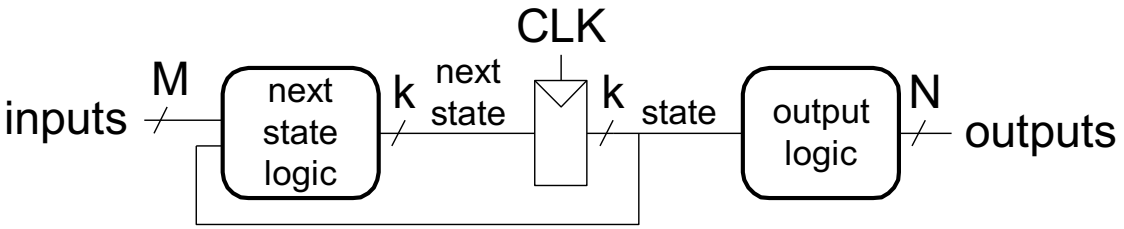
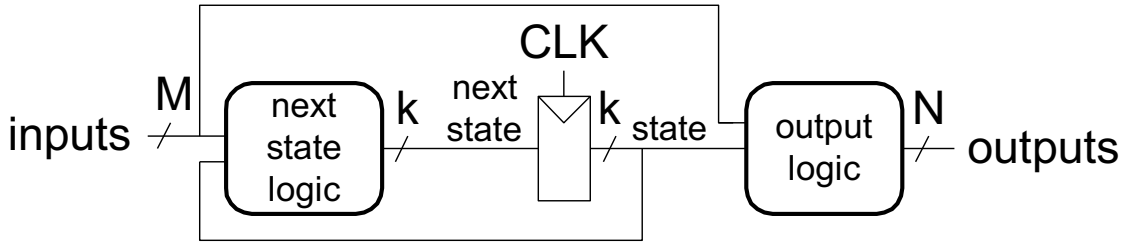


Moore FSM

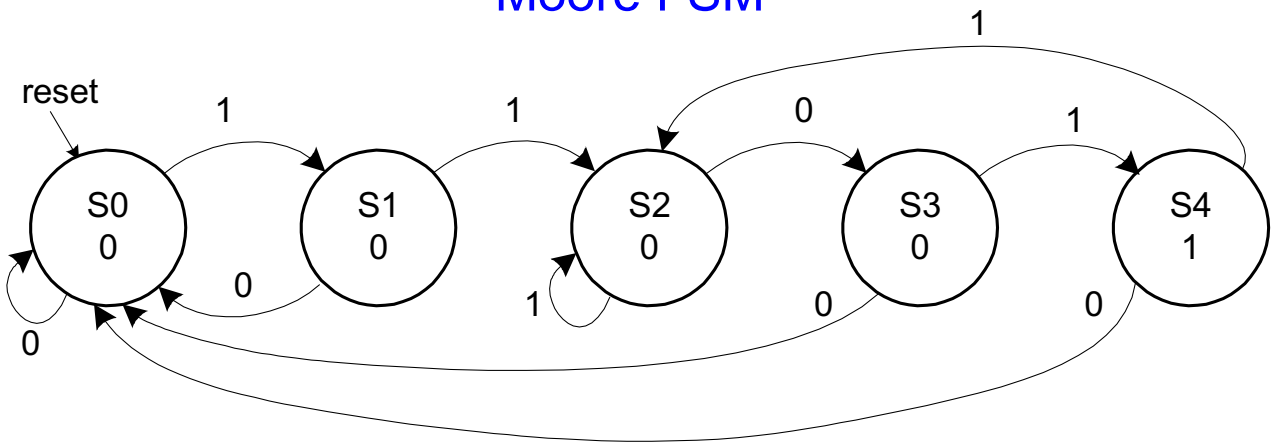


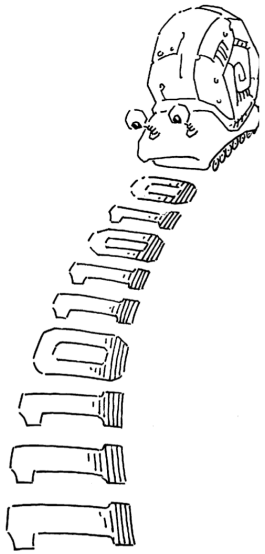
Mealy FSM



State Transition Diagrams

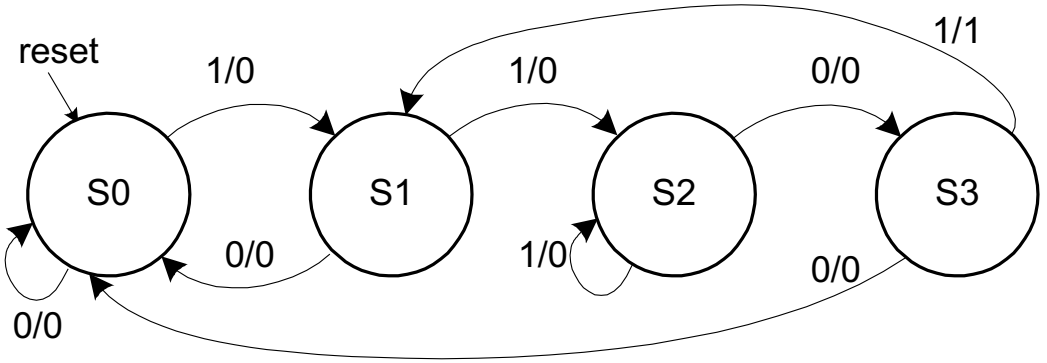
Moore FSM





What are the tradeoffs?

Mealy FSM



FSM Design Procedure

- **Determine** all possible states of your machine
- **Develop** a **state transition diagram**
  - Generally this is done from a textual description
  - You need to 1) determine the **inputs** and **outputs** for each **state** and 2) figure out how to get from one state to another
- **Approach**
  - Start by defining the **reset state** and what happens from it – this is typically an easy point to start from
  - Then continue to add **transitions** and **states**
  - Picking **good state names** is very important
  - Building an FSM is **like** programming (but it *is not* programming!)
    - An FSM has a sequential "control-flow" like a program with conditionals and goto's
    - The if-then-else construct is controlled by one or more inputs
    - The outputs are controlled by the state or the inputs
  - In hardware, we typically have many concurrent FSMs

What is to Come: LC-3 Processor

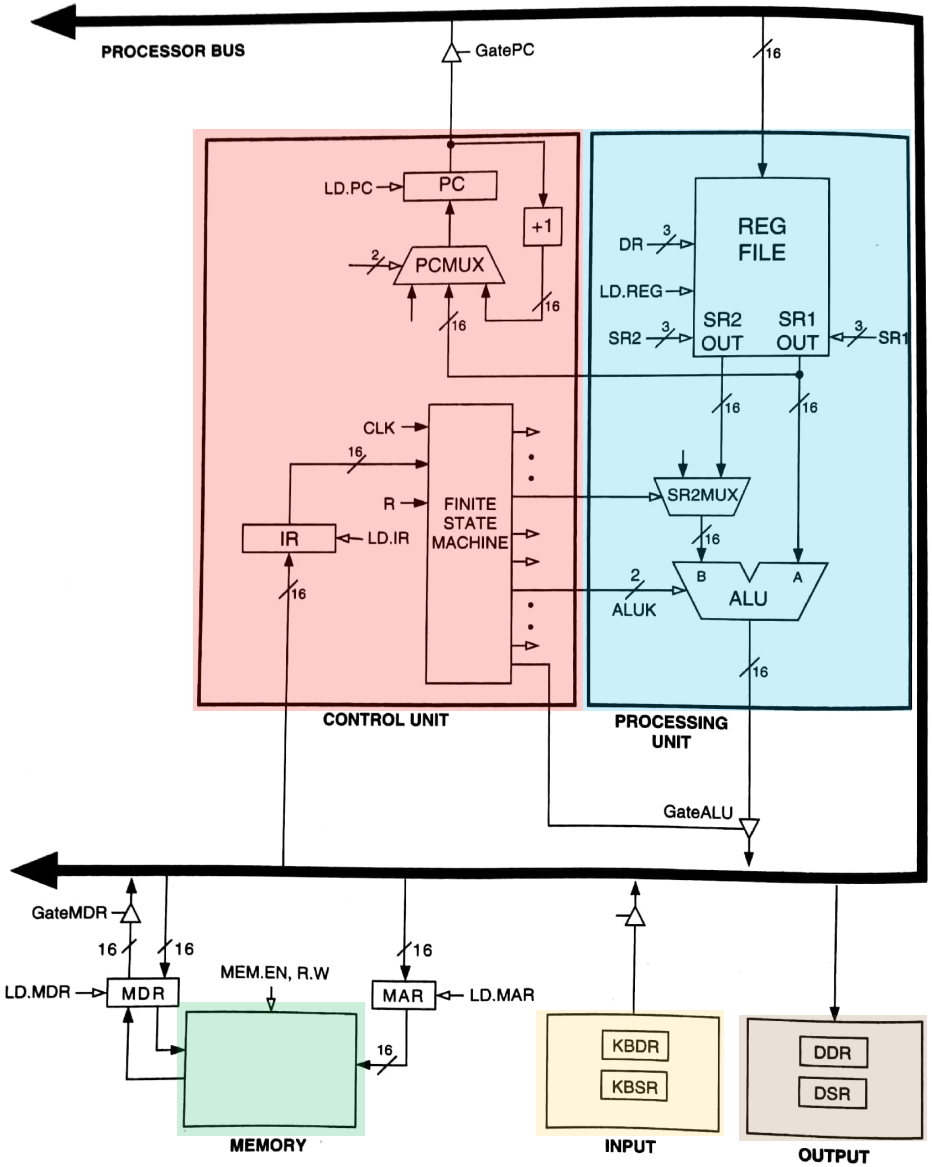
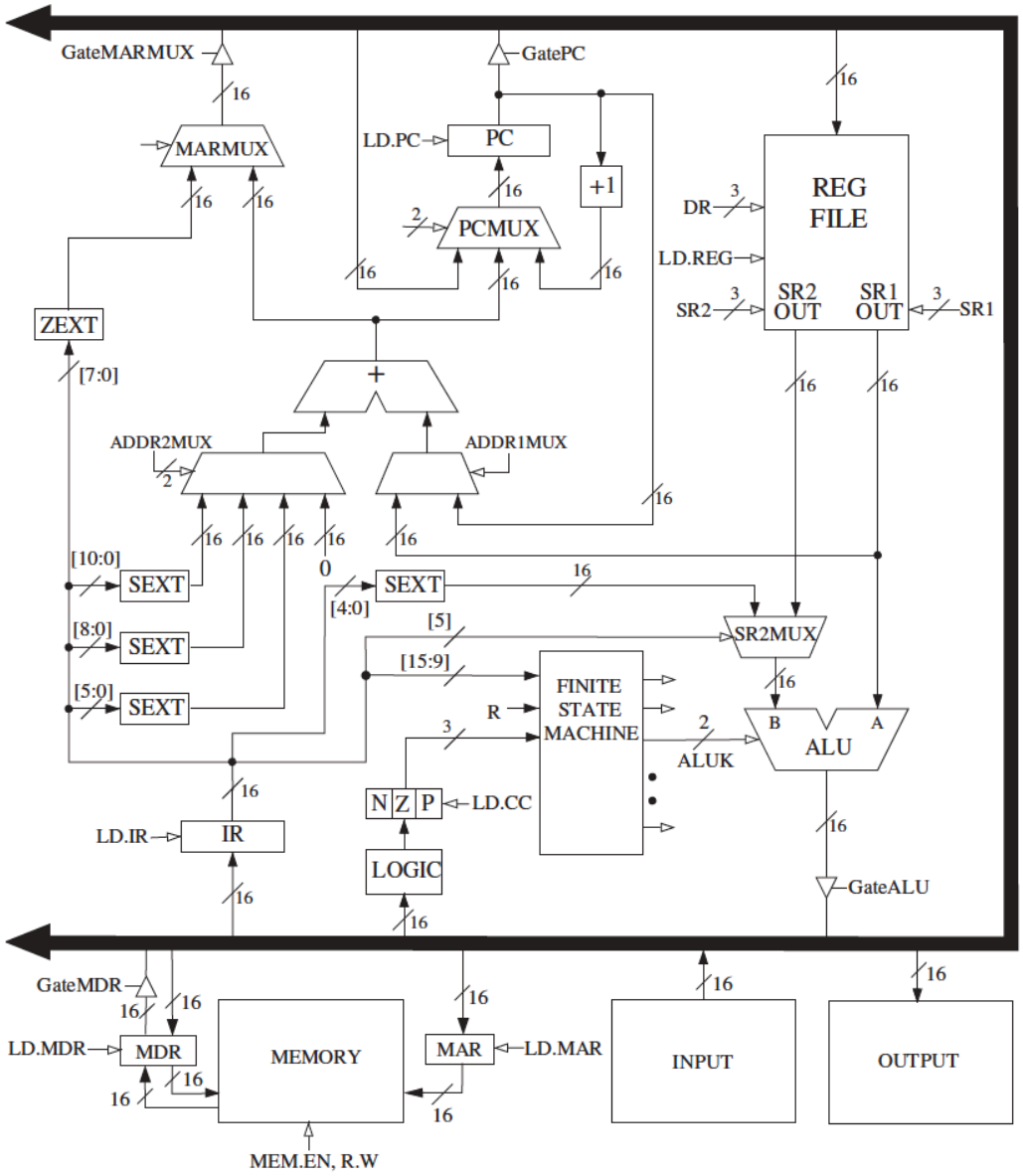


Figure 4.3 The LC-3 as an example of the von Neumann model

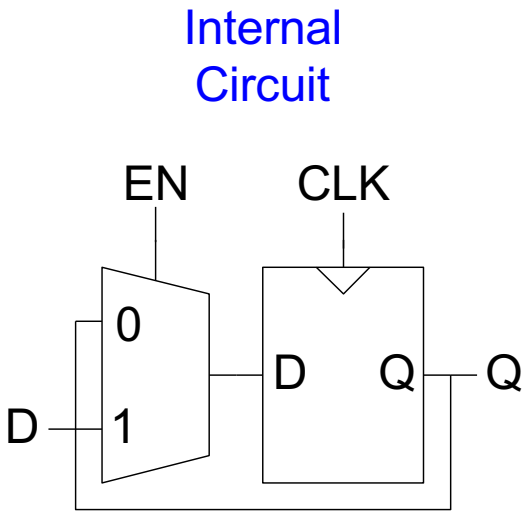
What is to Come: LC-3 Datapath



Backup Slides: Different Types of Flip Flops

Enabled Flip-Flops

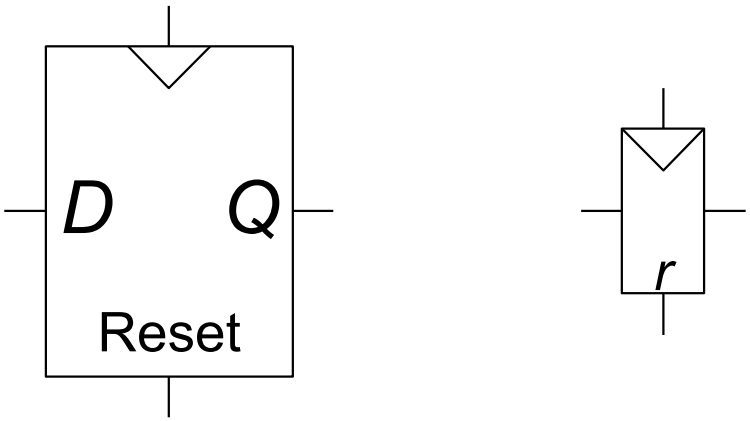
- **Inputs:** CLK, D, EN
  - The enable input (EN) controls when new data (D) is stored
- **Function:**
  - **EN = 1**: D passes through to Q on the clock edge
  - **EN = 0**: the flip-flop retains its previous state



Resettable Flip-Flop

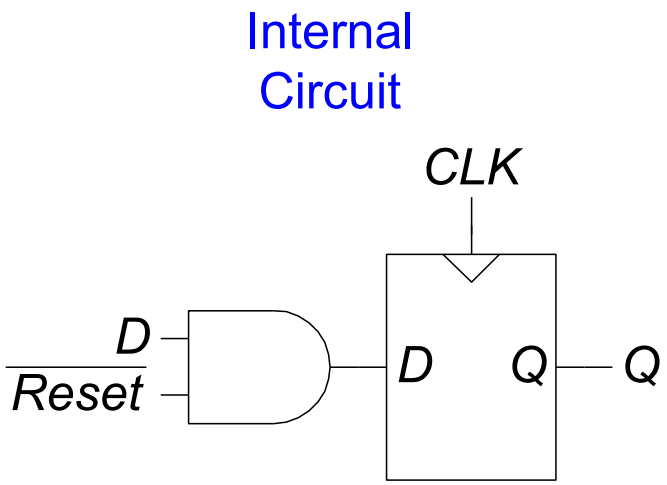
- **Inputs:** CLK, D, Reset
  - The Reset is used to set the output to 0.
- **Function:**
  - ***Reset = 1:*** Q is forced to 0
  - ***Reset = 0:*** the flip-flop behaves like an ordinary D flip-flop

Symbols



Resettable Flip-Flops

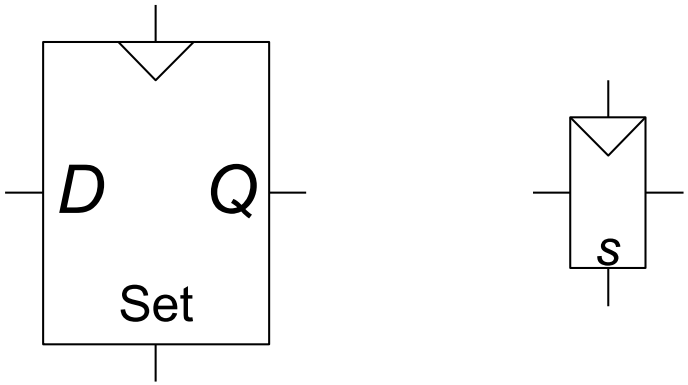
- Two types:
  - **Synchronous**: resets at the clock edge only
  - **Asynchronous**: resets immediately when Reset = 1
- Asynchronously resettable flip-flop requires changing the internal circuitry of the flip-flop (see Exercise 3.10)
- Synchronously resettable flip-flop?



Settable Flip-Flop

- **Inputs:** CLK, D, Set
- **Function:**
  - **Set = 1**: Q is set to 1
  - **Set = 0**: the flip-flop behaves like an ordinary D flip-flop

Symbols



Logic Simplification:
Karnaugh Maps (K-Maps)

Logic Simplification

- Systematic techniques for simplifications
  - amenable to automation

Key Tool: The Uniting Theorem — $F = A\overline{B} + AB$

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

$$F = A\overline{B} + AB = A(\overline{B} + B) = A(1) = A$$

Essence of Simplification:

Find two element subsets of the ON-set where only one variable changes its value. This single varying variable *can be eliminated!*

value is not needed

→ *B is eliminated, A remains*

A	B	G
0	0	1
0	1	0
1	0	1
1	1	0

$$G = \overline{A}\overline{B} + A\overline{B} = (\overline{A} + A)\overline{B} = \overline{B}$$

B's value stays the same within the ON-set rows

A's value changes within the ON-set rows

→ *A is eliminated, B remains*

Complex Cases

- One example

$$Cout = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

- Problem
  - Easy to see how to apply Uniting Theorem…
  - Hard to know if you applied it in all the right places…
  - …especially in a function of many more variables

- Question
  - Is there an easier way to find potential simplifications?
  - i.e., potential applications of Uniting Theorem…?

- Answer
  - Need an intrinsically *geometric* representation for Boolean f( )
  - Something we can draw, see…

Karnaugh Map

- Karnaugh Map (K-map) method
  - K-map is an alternative method of representing the truth table that helps visualize adjacencies in up to 6 dimensions
  - Physical adjacency ↔ Logical adjacency

2-variable K-map

A \ B	0	1
0	00	01
1	10	11

3-variable K-map

A \ BC	00	01	11	10
0	000	001	011	010
1	100	101	111	110

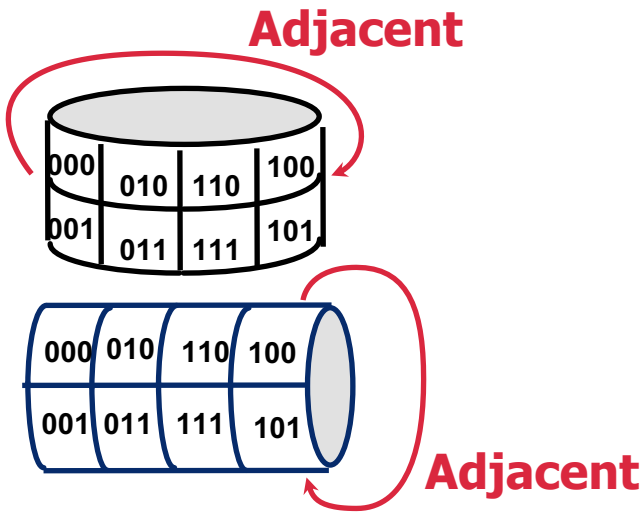
4-variable K-map

AB \ CD	00	01	11	10
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010

Numbering Scheme: **00, 01, 11, 10 is called a "Gray Code" — only a *single bit (variable) changes* from one code word and the next code word**

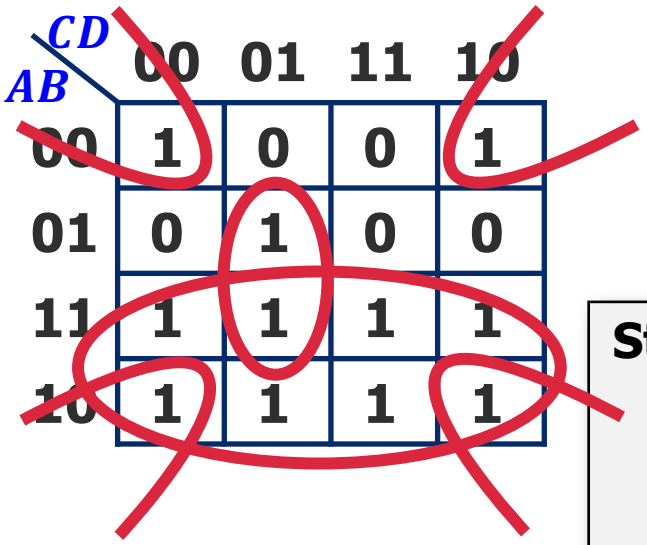
Karnaugh Map Methods

BC / A	00	01	11	10
0	000	001	011	010
1	100	101	111	110



K-map adjacencies go "around the edges"
Wrap around from first to last column
Wrap around from top row to bottom row

K-map Cover - 4 Input Variables



$$F(A,B,C,D) = \sum m(0,2,5,8,9,10,11,12,13,14,15)$$

$$F = A + \bar{B}\bar{D} + B\bar{C}D$$

Strategy for "circling" rectangles on Kmap:

Biggest "oops!" that people forget:

Logic Minimization Using K-Maps

- Very simple guideline:
  - Circle all the rectangular blocks of 1's in the map, using the fewest possible number of circles
    - Each circle should be as large as possible
  - Read off the implicants that were circled

- More formally:
  - A Boolean equation is minimized when it is written as a sum of the fewest number of prime implicants
  - Each circle on the K-map represents an implicant
  - The largest possible circles are prime implicants

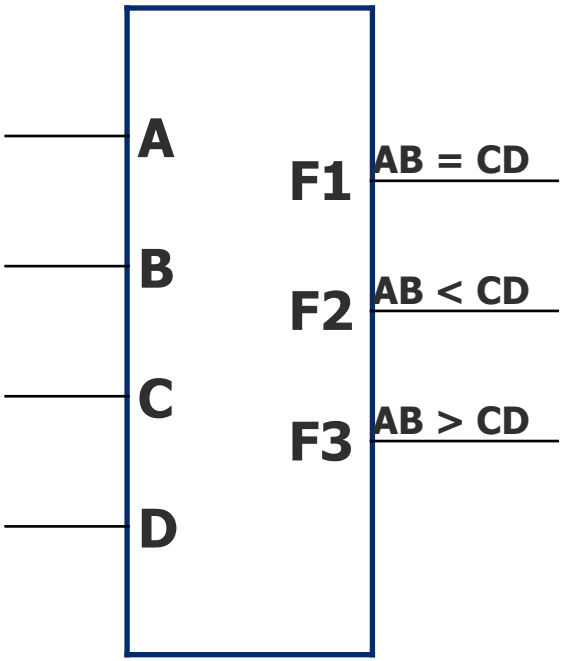
K-map Rules

- **What can be legally combined (circled) in the K-map?**
  - Rectangular groups of size 2^k for any integer k
  - Each cell has the same value (1, for now)
  - All values must be adjacent
    - Wrap-around edge is okay

- **How does a group become a term in an expression?**
  - Determine which literals are constant, and which vary across group
  - Eliminate varying literals, then AND the constant literals
    - constant 1 → use X, constant 0 → use $\overline{X}$

- **What is a good solution?**
  - Biggest groupings → eliminate more variables (literals) in each term
  - Fewest groupings → fewer terms (gates) all together
  - OR together all AND terms you create from individual groups

K-map Example: Two-bit Comparator

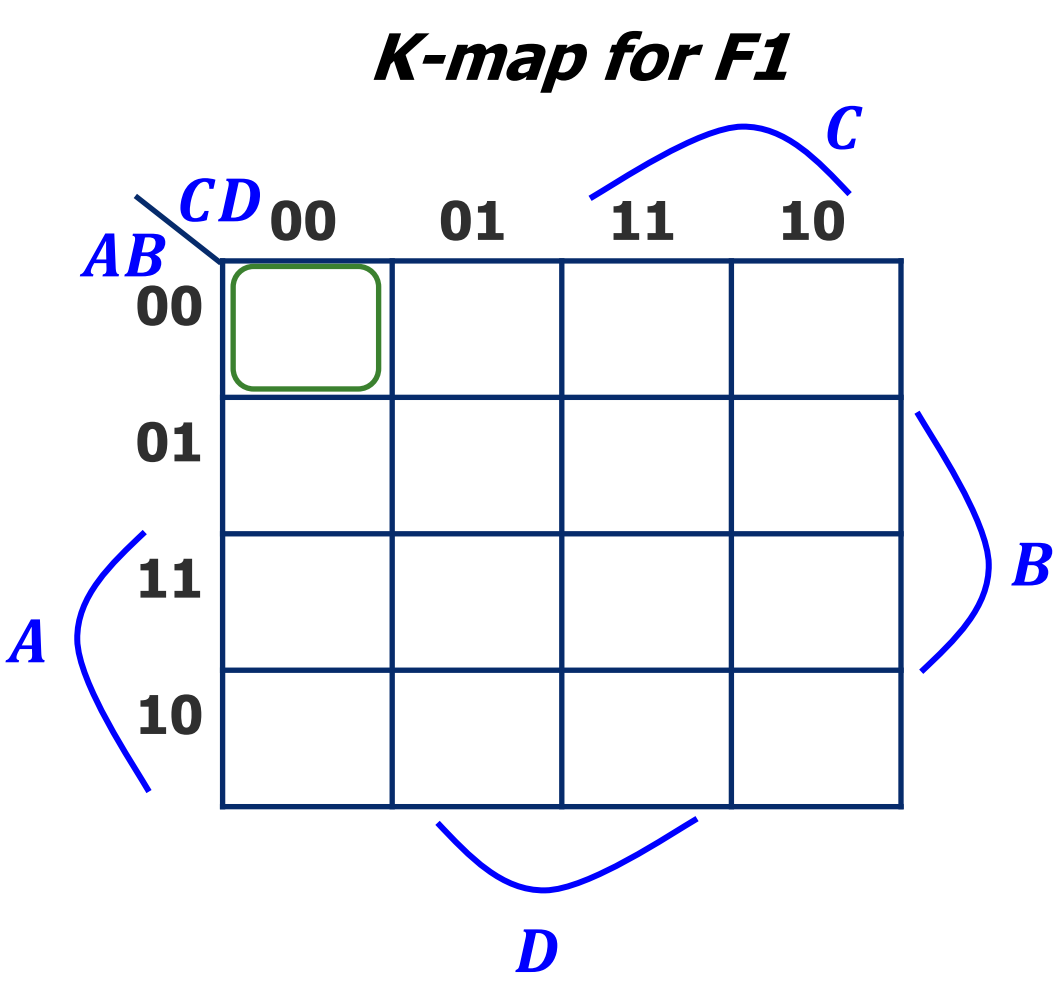


A	B	C	D	F1	F2	F3
0	0	0	0	1	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	0
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	1	0	0

Design Approach:

Write a 4-Variable K-map for each of the 3 output functions

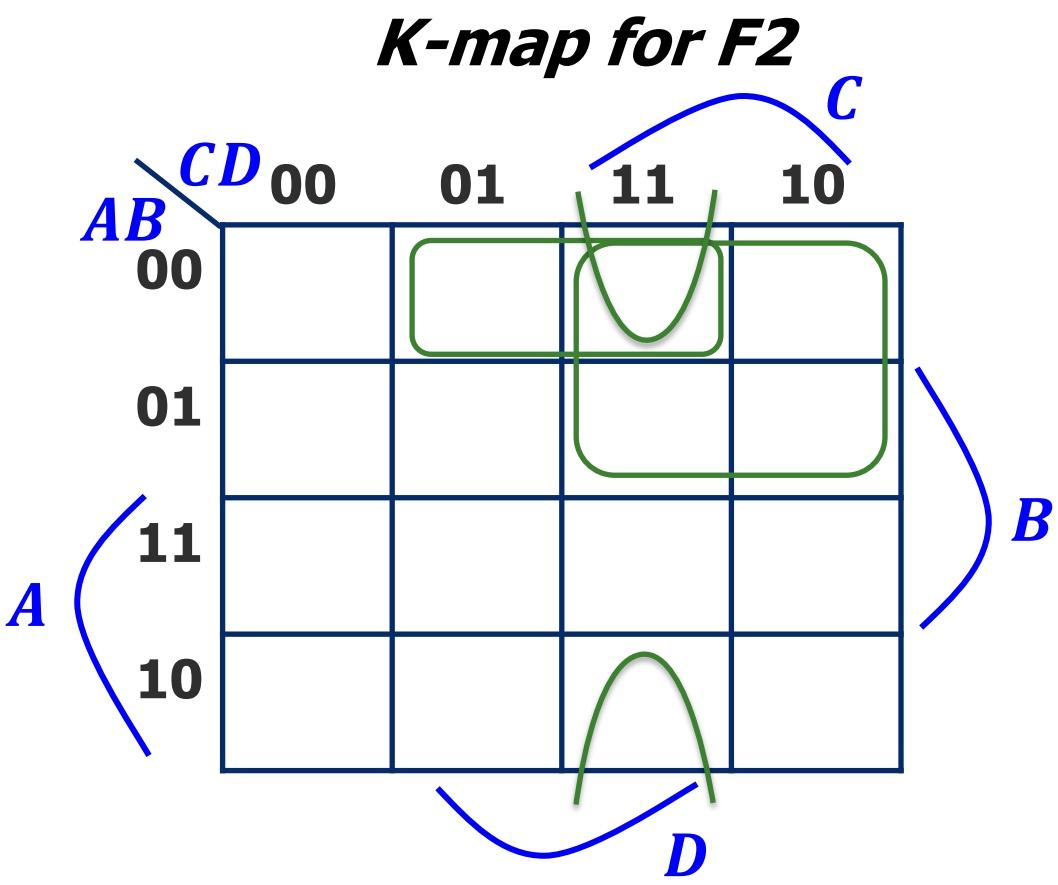
K-map Example: Two-bit Comparator (2)



F1 =

A	B	C	D	F1	F2	F3
0	0	0	0	1	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	0
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	1	0	0

K-map Example: Two-bit Comparator (3)



A	B	C	D	F1	F2	F3
0	0	0	0	1	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	0
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	1	0	0

F2 =

F3 = ? (Exercise for you)

K-maps with "Don't Care"

- Don't Care really means *I don't care what my circuit outputs if this appears as input*
  - You have an engineering choice to use DON'T CARE patterns intelligently as 1 or 0 to better simplify the circuit

A	B	C	D	F	G
...					
0	1	1	0	X	X
0	1	1	1		
1	0	0	0	X	X
1	0	0	1		
...					

Example: BCD Increment Function

- BCD (Binary Coded Decimal) digits
  - Encode decimal digits 0 - 9 with bit patterns 0000_2 — 1001_2
  - When incremented, the decimal sequence is 0, 1, …, 8, 9, 0, 1

A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

These input patterns should never be encountered in practice (hey -- it's a BCD number!) So, associated output values are "Don't Cares"

K-map for BCD Increment Function

A B

\+

W X

Z (without don't cares) =

Z (with don't cares) =

10	1		X	X	10			X	X

Y

AB \ CD	00	01	11	10
00		1		1
01		1		1
11	X	X	X	X
10			X	X

Z

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	X	X	X	X
10	1		X	X

A B C D

K-map Summary

- Karnaugh maps as a formal systematic approach for logic simplification
- 2-, 3-, 4-variable K-maps
- K-maps with "Don't Care" outputs
- H&H Section 2.7